1) $\sum_{k=2}^{n} (2k-4) = (n-2)(n-1)$

2) $9^n + 7$ ist durch 8 teilbar $n \in \mathbb{N}$

3) $a_n = 5 + 3/n$ $n \in \mathbb{N}$

4) $a_{n+1} = a_n^2 + 1/4$; $a_1 = 0$

1) $$\sum_{\underline{\underline{k=2}}}^{n} \underbrace{(2k-4)}_{a_k} = \underbrace{(n-2)(n-1)}_{S_n}$$

$n=2$ $\quad a_2 = S_2 \quad : \quad (2\cdot 2-4) = (2-2)(2-1)$

$$0 = 0 \quad \checkmark$$

$n+1$ $\quad S_{n+1} = S_n + a_{n+1}$

$$[(n+1)-2][(n+1)-1] = (n-2)(n-1) + [2(n+1)-4]$$

$$(n-1)\cdot(n) = n^2-3n+2 + (2n-2)$$

$$n^2-n = n^2-n$$

$$0 = 0 \quad \checkmark$$

2) $\boxed{9^n + 7} = 8 \cdot k \quad , \quad k \in \mathbb{Z}$

$n=1 \quad 9+7 = 16 = 8 \cdot 2 \quad ; \quad 2 \in \mathbb{Z}$ ✓

$n+1 \quad 9^{n+1} + 7 = 8 \cdot k$

$(9^n \cdot 9^1) + 7 = 8 \cdot k$

$\underbrace{9^n + 7} + 8 \cdot 9^n = 8K \rightarrow 8 \cdot \mathbb{Z} + 8 \cdot \mathbb{N} = 8 \cdot (\mathbb{Z} + \mathbb{N})$

$= 8 \cdot \mathbb{Z}$

$8 \cdot k_1 + 8 \cdot k_2 = 8 \cdot k$

β

$\mathbb{Z} + \mathbb{N} = \mathbb{Z}$ ✓

$\mathbb{Z}$

3) $a_n = 5 + 2/n$ $\qquad a_1 = 7$; $a_2 = 6$

<u>Monotonie:</u> $\qquad a_{n+1} < a_n$

$n = 1 \qquad a_2 < a_1 \iff 6 < 7$ ✓

$n+1 \qquad a_{n+2} < a_{n+1}$

$$5 + 2/(n+2) < 5 + 2/(n+1) \quad | -5 \quad | \cdot (n+1)(n+2)$$

$$2 \cdot (n+1) < 2 \cdot (n+2)$$

$$2n + 2 < 2n + 4$$

$$2 < 4$$ ✓

<u>Schranken</u> : $a_1 = 7$ ist obere Schranke

$a_n > 5$ (untere Schranke)

optimale

(Grenzwert)

$$\lim_{n \to \infty} (5 + 2/n) = \left[5 + \frac{2}{\infty}\right] = [5 + 0] = \underline{\underline{5}}$$

$a_n > 0$

$n = 1$ $\quad a_1 = 7 > 5$ ✓ $\qquad a_1 = 7 > 0$ ✓

$n+1$ $\quad a_{n+1} > 5$ $\qquad a_{n+1} > 0$

$5 + \frac{2}{n+1} > 5 \quad | -5$ $\qquad 5 + 2/(n+1) > 0 \quad | -5$

$2/(n+1) > 0 \quad | \cdot (n+1)$ $\qquad 2/(n+1) > -5 \quad | \cdot (n+1)$

$2 > 0$ ✓ $\qquad 2 > -5n - 5$

$-2/5 > -5n \to n > 2/25$ ✓

4) $a_{n+1} = a_n^2 + 1/4 \; ; \; a_1 = 0 \; ; \; a_2 = 1/4$

Monotonie: $a_{n+1} > a_n$

$n = 1 \qquad a_2 > a_1 \iff 1/4 > 0$ ✓

$n+1 \qquad a_{n+2} > a_{n+1}$

$$a_{n+1}^2 + 1/4 > a_n^2 + 1/4 \quad | -1/4 \quad \sqrt{}$$

$$a_{n+1} > a_n$$ ✓

Schranke: $a_1 = 0$ ist untere Schranke

Grenzwert: $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$

$\lim a_n = \beta$

$$\left.\right\} \quad \beta = \beta^2 + 1/4$$

$$\beta^2 - \beta + 1/4 = 0$$

$$\beta^2 - \beta + \tfrac{1}{4} = 0$$

$$\beta_{1/2} = \tfrac{1}{2} \pm \sqrt{\left(\tfrac{1}{2}\right)^2 - \tfrac{1}{4}} = \underline{\underline{\tfrac{1}{2}}}$$

<u>Schranke</u> $\quad a_n < \tfrac{1}{2}$

$n=1 \qquad a_1 = 0 < \tfrac{1}{2}$ ✓

$n+1 \qquad a_n < \tfrac{1}{2} \quad \left|\,(\,)^2\right.$

$$a_n^2 < \tfrac{1}{4} \quad \left|\, + \tfrac{1}{4}\right.$$

$$\underbrace{a_n^2 + \tfrac{1}{4}}_{} < \tfrac{1}{2}$$

$$a_{n+1} < \tfrac{1}{2}$$ ✓