1) $\dfrac{2-3i}{2-i} + \dfrac{4i}{1+3i}$ $\Big\}$ $z = a+bi$ ✓

2) $z^4 - (4i+8)z^3 + z^2 \cdot (16i - 13) = 0$ $\Big\}$ α

3) $\begin{pmatrix} 1 & x & -x \\ 3 & 1 & 3 \\ -2 & 4 & 1 \end{pmatrix} = A$; $\vec{b} = \begin{pmatrix} 9 \\ 3 \\ 9 \end{pmatrix}$

a) Wann gibt es A^{-1} ?

b) $A \cdot \vec{x} = \vec{b}$ $(x=2)$

4) $\vec{a} = (1;2,-1)^T$, $\vec{b} = (3;1;-1)$, $\vec{c} = (-2;3;2)$ $\Big\}$ e_1

$e_2 \Rightarrow 3x - 4y + 2z = 6$

1)

$$\frac{2-3i}{2-i}\cdot\frac{2+i}{2+i}=\frac{4+2i-6i-3i^2}{4-i^2}=\frac{7-4i}{5}$$

$$\frac{4i}{1+3i}\cdot\frac{1-3i}{1-3i}=\frac{4i-12i^2}{1-9i^2}=\frac{4i+12}{10}$$

$+$

$$\frac{14-8i+4i+12}{10}=\frac{26-4i}{10}=\frac{13}{5}-\frac{2}{5}i$$

$$r=\sqrt{\left(\frac{13}{5}\right)^2+\left(\frac{2}{5}\right)^2}=\sqrt{\frac{169}{25}+\frac{4}{25}}=\sqrt{\frac{173}{25}}$$

$$\alpha=\arctan\left(\frac{-2/5}{13/5}\right)+2\pi=\arctan\left(-2/13\right)+2\pi$$

2)

$$z^2 \cdot \left[z^2 + (-4i-8) \cdot z + (16i-13) \right] = 0$$

$$z_{1/2} = 0 \quad ; \quad r_{1/2} = 0 \quad ; \quad \alpha_{1/2} = 0$$

$$z_{3/4} = -\frac{-4i-8}{2} \pm \sqrt{\left(\frac{-4i-8}{2}\right)^2 - (16i-13)}$$

$$z_{3/4} = 2i+4 \pm \sqrt{(-2i-4)^2 - (16i-13)}$$

$$= 2i+4 \pm \sqrt{4i^2 + 16i + 16 - 16i + 13}$$

$$= 2i+4 \pm \sqrt{25} = 2i+4 \pm 5$$

$$z_3 = 2i+9 \quad ; \quad r_3 = \sqrt{85} \quad ; \quad \alpha_3 = \arctan(2/9)$$

$$z_4 = 2i-1 \quad ; \quad r_4 = \sqrt{5} \quad ; \quad \alpha_4 = \arctan(-2) + \pi$$

3) a) $\begin{pmatrix} 1 & x & -x \\ 3 & 1 & 3 \\ -2 & 4 & 1 \end{pmatrix} = \left.\begin{matrix} 1 - 6x - 12x \\ \ominus \\ 2x + 3x + 12 \end{matrix}\right\} \begin{matrix} 1 - 18x \\ \ominus \\ 5x + 12 \end{matrix}$

$\text{Det}(A) = -11 - 23x = 0 \qquad x = -{}^{11}/_{23}$

$x \in \mathbb{R} \setminus \{-{}^{11}/_{23}\}$, da somit regulär.

b) $x = 2 \qquad \begin{pmatrix} 1 & 2 & -2 \\ 3 & 1 & 3 \\ -2 & 4 & 1 \end{pmatrix} \cdot \vec{x} = \begin{pmatrix} 9 \\ 3 \\ 9 \end{pmatrix}$; $\text{Det}(A) = -57$

$x_1: \begin{pmatrix} 9 & 2 & -2 \\ 3 & 1 & 3 \\ 9 & 4 & 1 \end{pmatrix} = \begin{matrix} 9 + 54 - 24 \\ \ominus \\ -18 + 6 + 108 \end{matrix} = 39 - 96 = -57$

$-57/-57 = 1 = x_1$

$$x_2 \Rightarrow \begin{pmatrix} 1 & 9 & -2 \\ 3 & 3 & 3 \\ -2 & 9 & 1 \end{pmatrix} = 3 \cdot \begin{vmatrix} 1 & 3 & -2 \\ 3 & 1 & 3 \\ -2 & 3 & 1 \end{vmatrix} = 3 \cdot \begin{matrix} 1 - 18 - 18 \\ - \\ 4 + 9 + 9 \end{matrix}$$

$$= 3 \cdot (-35 - 22) = \frac{-3 \cdot 57}{-57} = \underline{\underline{+3}}$$

$$x_3 \Rightarrow 3 \cdot \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ -2 & 4 & 3 \end{pmatrix} = 3 \cdot \begin{matrix} 3 - 4 + 36 \\ - \\ -6 + 18 + 4 \end{matrix}$$

$$= 3 \cdot (35 - 16) = \frac{57}{-57} = -1$$

$$\vec{x} = (1; 3; -1)^T$$

4) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$; $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$; $\vec{c} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$

$$e_1 : \vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 3 - 1 \\ 1 - 2 \\ -1 + 1 \end{pmatrix} + \gamma \begin{pmatrix} -2 - 3 \\ 3 - 1 \\ 2 + 1 \end{pmatrix}$$

$$e_1 : \vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -5 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{array}{cc} \cancel{2} & \cancel{-5} \\ -1 & 2 \\ 0 & 3 \\ 2 & -5 \\ -1 & 2 \\ \cancel{0} & \cancel{3} \end{array} \qquad \left.\begin{array}{c} -3 - 0 \\ 0 - 6 \\ 4 - 5 \end{array}\right\} \vec{n} = \begin{pmatrix} -3 \\ -6 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix}$$

$$\left.\begin{array}{l} ax + by + cz = d \\ 3x + 6y + 1z = d \\ 3 \cdot (1) + 6 \cdot (2) + 1 \cdot (-1) = 14 \end{array}\right\} 3x + 6y + z = 14$$

$e_1 = 3x + 6y + z = 14$

$e_2 = 3x - 4y + 2z = 6$ $\Rightarrow$ Schnittgerade

$\vec{n_1} \times \vec{n_2} \stackrel{!}{=}$ Richtungsvektor der Schnittgeraden

$$\begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 + 4 \\ 3 - 6 \\ -12 - 18 \end{pmatrix} = \begin{pmatrix} 16 \\ -3 \\ -30 \end{pmatrix}$$

$$\left| \begin{array}{l} 3x + 6y + z = 14 \\ 3x - 4y + 2z = 6 \end{array} \right| \qquad y = 0 \qquad \left| \begin{array}{l} 3x = 14 + z \\ 3x = 6 - 2z \end{array} \right|$$

$14 - z = 6 - 2z \quad | + 2z - 14 \quad \Rightarrow z = -8$

$3x = 6 - 2 \cdot (-8) = 22 \qquad x = 22/3$

$$\vec{x} = \begin{pmatrix} 22/3 \\ 0 \\ -8 \end{pmatrix} + \beta \begin{pmatrix} 16 \\ -3 \\ -30 \end{pmatrix}$$