1) $a_{n+1} = -\frac{1}{3}\ (3-2a_n)$; $a_1 = 3$

2) $a_{n+1} = \left(\frac{a_n}{3}\right)^3 + 2$; $a_1 = 0$

3) $a_{n+1} = \sqrt{\frac{1}{2}a_n^2 + 1}$; $a_1 = 4$

2) $a_1 = 0$ $\quad a_2 = \left(\frac{a_1}{3}\right)^3 + 2 = 0 + 2 = 2$

Behauptung: $\quad a_{n+1} > a_n$

$n = 1$ $\quad a_2 > a_1 \Rightarrow 2 > 0$ ✓

Voraussetzung ...

$n+1$

$$a_{n+2} > a_{n+1}$$

$$\left(\frac{a_{n+1}}{3}\right)^3 + 2 > \left(\frac{a_n}{3}\right)^3 + 2 \quad | -2$$

$$\left(\frac{a_{n+1}}{3}\right)^3 > \left(\frac{a_n}{3}\right)^3 \quad | \sqrt[3]{}$$

$$\frac{a_{n+1}}{3} > \frac{a_n}{3} \quad | \cdot 3$$

$$a_{n+1} > a_n \quad \checkmark$$

Schranken: $a_1 = 8$ ist untere Schranke

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n = \beta$$

$$\left(\frac{\beta}{3}\right)^3 + 2 = \beta$$

$$\frac{\beta^3}{27} + 2 - \beta = 0 \qquad | \cdot 27$$

$$\beta^3 - 27\beta + 54 = 0 \qquad \beta = 3 \rightarrow 0$$

$$(\beta^3 - 27\beta + 54)(\beta - 3) = \underbrace{\beta^2 + 3\beta - 18}$$

$$-(\beta^3 - 3\beta^2)$$

$$3\beta^2 - 27\beta + 54$$

$$-(3\beta^2 - 9\beta)$$

$$-18\beta + 54$$

$$-(-18\beta + 54)$$

$$\beta_{1/2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 18}$$

$$-\frac{3}{2} \pm \sqrt{\frac{81}{4}}$$

$$\beta_1 = -\frac{3}{2} + \frac{9}{2} = 3$$

$$\beta_2 = -\frac{3}{2} - \frac{9}{2} = -6$$

Schranke: Behauptung $a_n < 3$

$n=1$ $\quad a_1 < 3 \quad \Rightarrow \quad 2 < 3$ ✓

$n+1$ $\quad a_n < 3 \quad | \cdot 1/3$

$$\frac{a_n}{3} < 1 \quad | \uparrow^3$$

$$\left(\frac{a_n}{3}\right)^3 < 1^3 = 1 \quad | +2$$

$$\underbrace{\left(\frac{a_n}{3}\right)^3 + 2}_{} < 3$$

$$a_{n+1} < 3$$ ✓