1)

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & y \\ 1 & -2 & 3 & -4 \\ 2 & x & 1 & -8 \end{array}\right) \quad \begin{array}{l} \\ |\cdot(-1) \\ \\ \end{array} \quad \begin{array}{l} \\ |\cdot(-2) \\ \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -1 & y+4 \\ 0 & x+4 & -5 & 0 \end{array}\right| \quad \begin{array}{l} \\ |\cdot(x+4) \\ |\cdot(-4) \end{array} \quad \begin{array}{l} \\ |\cdot(-5) \\ \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -1 & y+4 \\ 0 & x-16 & 0 & -5y-20 \end{array}\right|$$

$$\begin{array}{ccc|c} 0 & 0 & -x+16 & (y+4)(x+4) \\ & & & xy+4x+4y+16 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -1 & y+4 \\ 0 & x-16 & 0 & -5y-20 \end{array}\right|$$

I. $x <> 16 \;;\; y \in \mathbb{R}$ (Zahl | egal) → eindeutige Lösung

$$det(A) = \begin{vmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \\ 0 & 0 & x-16 \end{vmatrix} = -x+16 <> 0$$

$$Rg(A) = 3 \;\hat{=}\; \text{Maximalrang} \Rightarrow Rg(A|b) = 3$$

II. $x = 16 \;\wedge\; y <> -4$ (0 | Zahl) → keine Lösung

$$det(A) = \begin{vmatrix} 1 & -2 \\ 0 & 4 \end{vmatrix} = 4 <> 0 \Rightarrow Rg(A) = 2$$

$$det(A|b) = \begin{vmatrix} 1 & -2 & -4 \\ 0 & 4 & y+4 \\ 0 & 0 & -5y-20 \end{vmatrix} = 4\cdot(-5y-20) <> 0 \Rightarrow Rg(A|b) = 3$$

III.

$x = 16 \wedge y = -4$ $(0 \mid 0)$

$$\left|\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

$\det(A) = \begin{vmatrix} 1 & -2 \\ 0 & 4 \end{vmatrix} = 4 <> 0 \Rightarrow Rg(A) = 2 = Rg(A|b)$

, da Maximalrang

$$\left| \begin{array}{l} a - 2b + 3c = -4 \\ \quad 4b - c = 0 \end{array} \right|$$

$b = r$
$c = 4r$

$a - 2r + 12r = -4$
$a = -4 - 10r$

$$\vec{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -4 - 10r \\ r \\ 4r \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} -10 \\ 1 \\ 4 \end{pmatrix}$$