

$$(\mathbb{Z}, +) : \quad \underline{\text{Sinn}} : \quad \mathbb{Z} + \mathbb{Z} \rightarrow \mathbb{Z}$$

$$\begin{array}{l} \underline{\text{ass0}} : \quad (a + b) + c = a + (b + c) \\ \quad \quad \quad a + b + c = a + b + c \end{array} \left. \vphantom{\begin{array}{l} (a + b) + c = a + (b + c) \\ a + b + c = a + b + c \end{array}} \right\} \begin{array}{l} a, b, c \in \mathbb{Z} \\ \text{Standard} \\ \text{addition} \end{array}$$

$$\underline{\text{kom}} : \quad a + b = b + a$$

$$\underline{\text{neutral}} : \quad a + \mathbb{1} = a \\ \quad \quad \quad \mathbb{1} = 0 \in \mathbb{Z} \quad \checkmark$$

$$\underline{\text{invers}} : \quad a + \bar{a} = \mathbb{1} = 0 \\ \quad \quad \quad \bar{a} = -a \in \mathbb{Z} \quad \checkmark$$

\Rightarrow abelsche Gruppe

$$(\mathbb{Z} \setminus \{0\}, \cdot) : \underline{\text{G.r.d.}} : \mathbb{Z} * \mathbb{Z} \rightarrow \mathbb{Z}$$

neutral zu +

Standard-
multiplikation

$$\underline{\text{asso}} : (a * b) * c = a * (b * c) \\ a * b * c = a * b * c$$

$$\underline{\text{kom}} : a * b = b * a \quad (a, b, c \in \mathbb{Z})$$

$$\underline{\text{neutral}} : a * 1 = a \\ 1 = 1 \in \mathbb{Z} \quad \checkmark$$

$$\underline{\text{invers}} : a * \bar{a} = 1 = 1$$

$$\bar{a} = 1/a \in \mathbb{Z}$$

\Rightarrow abelsche Monoid

Distributiv : links : $a * (b + c) = ab + ac$
rechts : $(b + c) * a = ba + ca$ } =, da
komm.

Neutralelement : $a * 1_+ = 1_+$

=> abstr. , un.ä. Ring

2) Binär: $\{0,1\} + \{0,1\} \rightarrow \{0,1\}$

asso + kom: Binäre Standardaddition
auf einer Stelle

neutral: $a + 1 = a$ $a = 0 \in \{0,1\}$

invert:
$$\left. \begin{array}{l} 0 + \bar{a} = 1 = 0 \\ 1 + \bar{a} = 1 = 0 \end{array} \right\} \bar{a} = a$$

$(\{0,1\} \setminus \{0\}, *)$: Binär: $\{0,1\} * \{0,1\} \rightarrow \{0,1\}$

asso, kom: Binäre Multiplikation

neutral: $a * 1 = a$
 $a = 1 \in \{0,1\}$

invert: $\bar{a} = a$
 $a * \bar{a} = 1$