Mathe-Party 2019

Stabil:

$$42^{n+p} \quad ; \; n \in \mathbb{Z} \; ; \; p \in \mathbb{N} \qquad *$$

$$42^{1} = 42^{-3+4} = 42^{1+0}$$

$$42^{n_1+p_1} * 42^{n_2+p_2} = 42^{n_1+p_1+n_2+p_2}$$

$$= 42^{\underbrace{(n_1+n_2)}_{42^{n_3}} + \underbrace{(p_1+p_2)}_{p_3}} \quad \left. \begin{matrix} n_i \in \mathbb{Z} \\ p_i \in \mathbb{N} \end{matrix} \right.$$

associativ:

$$\left(42^{n_1+p_1} * 42^{n_2+p_2}\right) * 42^{n_3+p_3} = 42^{n_1+p_1} * \left(42^{n_2+p_2} * 42^{n_3+p_3}\right)$$

$$42^{(n_1+n_2)+(p_1+p_2)} * 42^{n_3+p_3} = 42^{n_1+p_1} * 42^{(n_2+n_3)+(p_2+p_3)}$$

$$42^{(n_1+n_2+n_3)+(p_1+p_2+p_3)} = 42^{(n_1+n_2+n_3)+(p_1+p_2+p_3)}$$

$$0 = 0$$

Kommutativ

$$42^{n_1+p_1} * 42^{n_2+p_2} = 42^{n_2+p_2} * 42^{n_1+p_1}$$

$$42^{(n_1+n_2)+(p_1+p_2)} = 42^{(n_1+n_2)+(p_1+p_2)}$$

$$0 = 0$$

neutral:

$$42^{n+p} * 42^{n_1+p_1} = 42^{n+p}$$

$$42^{(n+n_1)+(p+p_1)} = 42^{n+p}$$

$$n_1 = 0 \wedge p_1 = 0 \in \mathbb{N}$$

$$\in \mathbb{Z}$$

$$1 = 42^{0+0}$$

Inverse

$$42^{n+p} * 42^{\bar{n}+\bar{p}} = 42^{0+0}$$

$$42^{\underbrace{(n+\bar{n})}_{0} + \underbrace{(p+\bar{p})}_{0}} = 42^{0+0}$$

$$\bar{n} = -n \in \mathbb{Z} \ ; \ \bar{p} = -p \in \mathbb{Z}$$

$\Rightarrow$ abelsches Monoid

2)
$$\left(\begin{array}{ccc|c} 2 & 1 & -2 & 0 \\ 1 & 4 & 4 & 0 \\ 5 & 1 & 1 & 0 \end{array}\right) \quad | \cdot (-2) \quad | \cdot (-5)$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 4 & 0 \\ 0 & -7 & -10 & 0 \\ 0 & -14 & -14 & 0 \end{array}\right) \quad | : (-14)$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 4 & 0 \\ 0 & -7 & -10 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \quad 1 \cdot 7 \; +$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right) \Bigg\} \; \alpha = \beta = \gamma = 0$$

Triviallösung

$\Rightarrow$ linear unabhängig

3)

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} * \begin{pmatrix} \alpha \\ -2 \\ -4 \end{pmatrix} = \alpha + 2 - 8 = \alpha - 6 = 0$$

$$\alpha = 6$$

$$\left.\begin{array}{l} |\vec{a}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6} \\ |\vec{b}| = \sqrt{6^2 + (-2)^2 + (-4)^2} = \sqrt{56} \end{array}\right\} * \Rightarrow A = 18{,}33 \text{ FE}$$

5) a

$$E: \vec{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -1 - 2 \\ 0 + 1 \\ 1 - 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -3 - 2 \\ 1 + 1 \\ 2 - 0 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} + \beta \cdot \begin{pmatrix} -5 \\ 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -5 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 - 2 \\ -5 + 6 \\ -6 + 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$0 \cdot x + 1 \cdot y - 1 \cdot z = d$$

$$2 \qquad -1 \qquad 0 \qquad \Rightarrow +y - z = -1$$

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 - 3 \\ -2 + 0 \\ +0 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\left| \begin{array}{l} 2x - 3y - z = 5 \\ y - z = -1 \end{array} \right| \longrightarrow \begin{array}{l} y = 0 \\ z = 1 \\ x = 3 \end{array}$$

$$g: \vec{x} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

x-Achse : $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ y-Achse $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ z-Achse $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} * \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 2 = \sqrt{6} \cdot \sqrt{1} \cdot \cos(\alpha)$$

$$\alpha_x = \arccos 2/\sqrt{6}$$

$$\alpha_y = \arccos 1/\sqrt{6} = \alpha_z$$

7) $\operatorname{Det}(A) = \alpha + 8 \quad \Rightarrow \alpha <> -8$

$\Rightarrow \operatorname{Det}(A) <> 0$ regulär

$$A^{-1} = \begin{pmatrix} 7 & -12 & -31 \\ -8 & -14 & -35 \\ 3 & 5 & -13 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 8 & -3 \\ 12 & 14 & -5 \\ 31 & 35 & -13 \end{pmatrix}^T$$

9 $\quad |A - \lambda \cdot E| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ 6 & -1-\lambda & 0 \\ -1 & -2 & -1-\lambda \end{vmatrix} = 0$$

$$1 + 2\lambda + \lambda^2$$
$$-\lambda - 2\lambda^2 - \lambda^3$$

$$(1-\lambda)\cdot(-1-\lambda)^2 + 0 - 12 = (1-\lambda)(1+2\lambda+\lambda^2) - 12$$

$$-$$

$$-1(-1-\lambda) + 12\cdot(-1-\lambda) + 0 \qquad 1 + \lambda - 12 - 12\lambda$$

$$-\lambda^3 - \lambda^2 + \lambda - 11 - (-11 - 11\lambda)$$

$$-\lambda^3 - \lambda^2 + 12\lambda = -\lambda\,(\lambda^2 + \lambda - 12)$$
$$= -\lambda\,(\lambda+4)(\lambda-3)$$

$$\mathbb{L} = \{+3; -4, 0\}$$

10) a) $\det(A) = 11 \neq 0$ => regulär => invertierbar

b) $$A^{-1} = \frac{1}{11} \cdot \begin{pmatrix} -1 & 3 & -3 \\ -4 & 1 & 10 \\ 4 & -1 & 1 \end{pmatrix}$$

c) $\vec{x} = \left(\frac{17}{11} ; \frac{-42}{11} ; \frac{-2}{11} \right.$

WS 2018/19 5)

$$\left(\begin{array}{ccc|c} 1 & x & 5 & 4 \\ -1 & x & -1 & 0 \\ 1 & -2 & -1 & y \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & x & -1 & 0 \\ 0 & 2x & 4 & 4 \\ 0 & x-2 & -2 & y \end{array}\right) \quad | \cdot \frac{1}{2}$$

$$\left(\begin{array}{ccc|c} -1 & x & -1 & 0 \\ 0 & x & 2 & 2 \\ 0 & 2x-2 & 0 & y+2 \end{array}\right)$$

I $\quad 0 \mid 0$

$x = 1 \;\wedge\; y = -2$ $\quad \left\{ \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right) \right.$

$Det(A) = \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 <> 0$ Maximalrang = Rg

$\Rightarrow Rg(A) = 2 \qquad Rg(A|b) = 2$

$\Rightarrow \infty$ Lösungen

II $\quad 0 \mid <> 0 \qquad x = 1 \;\wedge\; y <> -2$

$Det(A) = \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 <> 0 \qquad Rg(A) = 2$

$\neq$

$Det(A|b) = -1 \cdot (y+2) <> 0 \qquad Rg(A|b) = 3$

$\Rightarrow$ keine Lösung

$\overline{\underline{III}}$ $\Leftrightarrow 0 \quad l \in \mathbb{R} \qquad x \Leftrightarrow 1 \quad \wedge \; y \in \mathbb{R}$

$$\left(\begin{array}{ccc|c} -1 & x & -1 & 0 \\ 0 & x & 2 & 2 \\ 0 & 2x-2 & 0 & y+1 \end{array}\right)$$

$\mathrm{Det}(A) = -2 \cdot (2x-2) \Leftrightarrow 0$

$Rg(A) = 3 \;\hat{=}\;$ Maximalrang $= Rg(A|B)$

$\Rightarrow$ eindeutig lösbar

$$\left|\begin{array}{ccccccc} -a & + & b & - & c & = & 0 \\ 0 & + & b & + & 2c & = & 2 \end{array}\right| \qquad \gamma = c$$

$$b = 2 - 2\gamma$$

$$a = (2 - 2\gamma) - \gamma = 2 - 3\gamma$$

$$\vec{x} = \begin{pmatrix} 2 - 3\gamma \\ 2 - 2\gamma \\ \gamma \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$$

- Alter der Tiere im Berliner Zoo in 2018

sachlich : Alter   Masse : alle Tiere
räumlich : Berliner Zoo   Einheit : 1 Tier
zeitlich : 2018

=> Intervallskala

15)

x_i	1	2	3	4	5	Σ
$h(x_i)$	5	4	3	4	4	20
$x_i \cdot h(x_i)$	5	8	9	16	20	58
$f(x_i)$	$5/20$	$4/20$	$3/20$	$4/20$	$4/20$	1
$F(x_i)$	$5/20$	$9/20$	$12/20$	$16/20$	1	

$\mu = 58/20 = 2{,}9$ $\qquad \overline{x}_Z = 1/2 \cdot (x_{10} + x_{11}) = 1/2 \cdot (3+3) = 3$

$Q_{0{,}25}: 0{,}25 \cdot 20 = 5 = x_5 = 1$

$Q_{0{,}75}: 0{,}75 \cdot 20 = 15 = x_{15} = 4$ $\qquad$ Modus $= 1$, da $h(1) = 5$ max.

						Σ
x_i	1	2	3	4	5	
$h(x_i)$	5	4	3	4	4	
$x_i^2 \cdot h(x_i)$	5	16	27	64	100	212
$\lvert x_i - \bar{x}_z \rvert$	2	1	0	1	2	
$\cdot h(x_i)$	10	4	0	4	8	26

$\mu = 2{,}9$

$\bar{x}_z = 3$

$\sigma^2 \to \sigma$

MAD

$$\sigma^2 = \tfrac{1}{20} \cdot 212 - 2{,}9^2 = 2{,}19$$

$$\sigma = \sqrt{2{,}19} = 1{,}48$$

$$VC = \frac{1{,}48}{2{,}9}$$

$$MAD = \tfrac{1}{20} \cdot 26 = 1{,}3$$

24) $K_0 = 8.500$ $\quad t_1 = \frac{196}{360}$ $\quad n = 7$ Jahre

$$13.460{,}16 = 8.500 \cdot \left(1 + 0{,}06 \frac{196}{360}\right) \cdot 1{,}06^7 \cdot \left(1 + 0{,}06 \cdot \frac{x}{360}\right)$$

$$1{,}0198 = 1 + 0{,}06 \cdot \frac{x}{360}$$

$x = 119{,}01$ Tage $\rightarrow$ 120 Tage

$\rightarrow$ 30.4 2013

25) $K_0 = 120.000$ $\quad K_7 = 2000{,}-$ $\quad p = 12\,\%$

$$K_4 = 120.000 \cdot 0{,}88^4 = 71.963{,}44$$

$$- 2.000{,}-$$

$$23.321{,}15 \xleftarrow{:3} 69.963{,}44$$

26) Maschine 250.000 15 Jahre
Restwert 1.000

a) Prozentsatz

b) Wie lange mit p aus a) um auf Restwert < 10.000

a) $p = \left(1 - \sqrt[15]{\frac{1.000}{250.000}}\right) = \underbrace{0,3079}_{30\,\%}$

b) $K_n = K_0 \cdot 0,7^n$

$10.000 = 250\,000 \cdot 0,7^n \quad | : 250.000$

$0,04 = 0,7^n$

$n = \log_{0,7} 0,04 = 9,02 \Rightarrow 10$ Jahre

$$n = \frac{\log 10.000 - \log 250.000}{\log (0{,}7)} = 9{,}02$$

28) $n = 6$ $\quad 4\,i.$ $\quad m = 12$ $\quad (m+1)$ $\quad 120 = r$

$$r_e = 120 \cdot \left(12 + \frac{0{,}04 \cdot (12+1)}{2} \right) = 1471{,}20$$

$$R_6 = 1471{,}20 \cdot \frac{1{,}04^6 - 1}{0{,}04} = 9758{,}43$$

$$R_{10} = 9758{,}43 \cdot 1{,}05^4 = 11.861{,}44$$

$$R_{10} = R_0 \cdot 1{,}04^{10} \qquad R_0 = \frac{11.861{,}44}{1{,}04^{10}}$$

$$R_0 = 8.013{,}16$$

29) $S = 125.000$ $n = 15$ $m = 6\ (m+1)$

$i = 0{,}07$

a) $A = 125.000 \cdot \dfrac{1{,}07^{16} - 1{,}07^{15}}{1{,}07^{15} - 1} = 13.724{,}33$

b) $a = \dfrac{13.724{,}33}{\left(6 + \dfrac{0{,}07 \cdot (6+1)}{2}\right)} = 2.197{,}65$

$Z_k = i \cdot RS_k - a \cdot \dfrac{i \cdot (m+1)}{2}$

$\underbrace{125.000 \cdot 0{,}07}_{-} - \underbrace{2197{,}65 \cdot \dfrac{0{,}07 \cdot 7}{2}}_{+}$

$Z_1 \Rightarrow 336{,}57$ $RS_2 = 125.000$
$- 5 \cdot 2197{,}65$
$- (2197{,}65 - 336{,}57)$

19) a) $P(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$

$P(X=4) = \binom{7}{4} \cdot 0{,}48^4 \cdot 0{,}52^3 = 0{,}26$

b) $P(X \leq 4) = 1 - P(X>4) = 80{,}5\,\%$

$P(X>4) = P(X=5) + P(X=6) + P(X=7)$

$$\left.\begin{aligned} P(X=5) &= \binom{7}{5} \cdot 0{,}48^5 \cdot 0{,}52^2 \\ P(X=6) &= \binom{7}{6} \cdot 0{,}48^6 \quad 0{,}52 \\ P(X=7) &= \binom{7}{7} \cdot 0{,}48^7 \cdot 0{,}52^0 \end{aligned}\right\} 19{,}5\,\%$$

$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$

20) $n = 100 \quad p = 0{,}05$

$$P(x > 7) = 1 - P(x \leq 7)$$

$$= 1 - 0{,}8720$$

$$= 0{,}128 = 12{,}8\,\%$$

7) Klausur: $w = 0{,}6$ $\quad m = 0{,}4$

	w	m
vegi	0,34	0,27
vegan	0,14	0,05
alles	0,52	0,68

	alles	vegi	vegan	
m	0,272	0,108	0,02	0,4
w	0,312	0,204	0,084	0,6
	0,584	0,312	$\underline{\underline{0{,}104}}$	1

abhängig

c) $P(M \cap \text{Vegan}) = P(M) \cdot P(\text{Vegan})$

$0{,}02 = 0{,}4 \cdot 0{,}104$

$\underline{0{,}02 = 0{,}0416}$

$\frac{0{,}204}{0{,}312} = 0{,}65$

$P(\text{Vegan}) = \underline{\underline{0{,}104}}$

$P_{\text{Vgi}}(W) = \frac{P(W \cap \text{Vegi})}{P(\text{Vegi})}$

g) $n = 3 \qquad m = 4 \ (m+1) \qquad q = 1{,}11 \qquad A = 51.300$

$$a = \frac{51.300}{\left(4 + \frac{0{,}11}{2} \cdot 5\right)} = 12.000$$

$$A = S \cdot \frac{q^{n+1} - q^{n}}{q^{n} - 1}$$

$$S = 51.300 \cdot \frac{1{,}11^{3} - 1}{1{,}11^{4} - 1{,}11^{3}} = 125.362{,}57$$

8) b)

$$49.261,40 = 5.627 \cdot 1,015 \cdot \frac{1,015^n - 1}{1,015 - 1}$$

$$R_n = r \cdot q \cdot \frac{q^n - 1}{q - 1}$$

$$\frac{\log\left(\frac{49.261,40 \cdot 0,015}{5627 \cdot 1,015} + 1\right)}{\log 1,015} = 8$$