| | | | | |
|---|---|---|---|---|
| | 120000 | | | |
| 1 | 109000 | 11000 | 9,17% | 9,17% |
| 2 | 98000 | 11000 | 10,09% | 9,17% |
| 3 | 87000 | 11000 | 11,22% | 9,17% |
| 4 | 76000 | 11000 | 12,64% | 9,17% |
| 5 | 65000 | 11000 | 14,47% | 9,17% |
| 6 | 54000 | 11000 | 16,92% | 9,17% |
| | | 11000 | | |

| | | | | |
|---|---|---|---|---|
| | 120000 | | | |
| 1 | 93600 | 26400 | 22% | 22,00% |
| 2 | 73008 | 20592 | 22% | 17,16% |
| 3 | 56946 | 16062 | 22% | 13,38% |
| 4 | 44418 | 12528 | 22% | 10,44% |
| 5 | 34646 | 9772 | 22% | 8,14% |
| 6 | 27024 | 7622 | 22% | 6,35% |
| | 27023,9521 | | | |

| | | | | |
|---|---|---|---|---|
| | 200000,00 | | | |
| 1 | 165000,00 | 35000,00 | 17,50% | 17,50% |
| 2 | 136125,00 | 28875,00 | 17,50% | 14,44% |
| 3 | 112303,13 | 23821,88 | 17,50% | 11,91% |
| 4 | 92650,08 | 19653,05 | 17,50% | 9,83% |
| 5 | 76436,31 | 16213,76 | 17,50% | 8,11% |
| 6 | 63059,96 | 13376,36 | 17,50% | 6,69% |
| 7 | 52024,47 | 11035,49 | 17,50% | 5,52% |
| 8 | 42920,18 | 9104,28 | 17,50% | 4,55% |
| 9 | 35409,15 | 7511,03 | 17,50% | 3,76% |
| 10 | 29212,55 | 6196,60 | 17,50% | 3,10% |
| 11 | 24100,35 | 5112,20 | 17,50% | 2,56% |
| 12 | 19882,79 | 4217,56 | 17,50% | 2,11% |
| 13 | 16403,30 | 3479,49 | 17,50% | 1,74% |
| 14 | 13532,73 | 2870,58 | 17,50% | 1,44% |
| 15 | 11164,50 | 2368,23 | 17,50% | 1,18% |
| 16 | 9210,71 | 1953,79 | 17,50% | 0,98% |
| 17 | 7598,84 | 1611,87 | 17,50% | 0,81% |
| 18 | 6269,04 | 1329,80 | 17,50% | 0,66% |
| 19 | 5171,96 | 1097,08 | 17,50% | 0,55% |
| 20 | 4266,87 | 905,09 | 17,50% | 0,45% |

2) ges: $n = 20$ Jahre

$p = 17{,}5\ \%$ => $q = 0{,}825$

$K_{20} = 4.266{,}87$

a) $K_0 = ?$ $K_{20} = K_0 \cdot q^n$

$K_0 = 200.000{,}-$

b) ... c) nach 12 Jahren (Excel)

$R_n (< 20.000) = 200.000 \cdot 0{,}825^n$

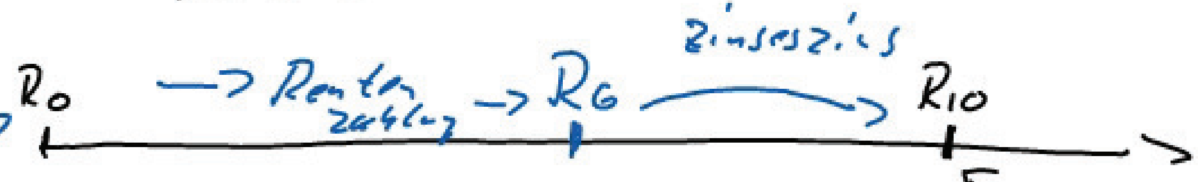
$1/10 = 0{,}825^n$

$n = \log_{0{,}825} 0{,}1 \approx 12$

1) $p = 8\,\%$  $q = 1{,}08$

$n = 10$ Jahre (nachschüssig)

$r = 12.500$




$$R_{10} = 12.500 \cdot \frac{1{,}08^{10} - 1}{0{,}08} = 181.082$$

$$R_0 \cdot q^n = R_{10} \qquad R_0 = \frac{181.082}{1{,}08^{10}} = 83.876$$

=> Laufzeit der Rentenzahlung 6 Jahre

$$R_6 \cdot q^4 = R_{10} \qquad R_6 = \frac{181.082}{1{,}08^4} = 133.100$$

$$r = R_6 \cdot \frac{0{,}08}{1{,}08^6 - 1} = 18.143{,}67$$

# Lösungen:

zu 1) $$R_{10} = 12.500 \cdot \frac{1{,}08^{10}-1}{1{,}08-1} = 181.083{,}03$$

$$R_0 = \frac{181.083{,}03}{1{,}08^{10}} = 83.876{,}48$$

zu 2) $$R_0 = r \cdot \frac{q^n-1}{q^{n+1}-q^n}$$

$$r = R_0 \cdot \frac{q^{n+1}-q^n}{q^n-1} = 10.380 \cdot \frac{1{,}05^{16}-1{,}05^{15}}{1{,}05^{15}-1} \approx 1.000$$

zu 3) $$R_n = r \cdot \frac{q^n-1}{q-1}$$

$$n = \frac{log\left[\frac{R_n}{r}\cdot(q-1)+1\right]}{log(q)} = \frac{log\left[\frac{39.792{,}82}{2.500}\cdot 0{,}05+1\right]}{\log(1{,}05)} = 12\ Jahre$$

# Lösungen:

zu 1) $R_n = r \cdot q \cdot \frac{q^n-1}{q-1} = r \cdot \frac{q^{n+1}-q}{q-1}$

$$q^n = \frac{R_n}{r \cdot q} \cdot (q-1) + 1$$

$$n = \frac{log\left[\frac{R_n}{r \cdot q} \cdot (q-1) + 1\right]}{\log(q)}$$

$$n \geq \frac{log\left[\frac{100.000}{1.000 \cdot 1{,}06} \cdot (1{,}06-1) + 1\right]}{\log(1{,}06)} = 32{,}54 \approx 33\ Jahre$$

zu 2) $R_{2016} = r \cdot \frac{q^{n+1}-q}{q-1} = 5.000 \cdot \frac{1{,}05^5-1{,}05}{1{,}05-1} = 22.628{,}16$

$$K_{2009} = R_{2016} \cdot q^{-n} = \frac{22.628{,}16}{1{,}05^3} = 19.547{,}05$$

# Lösung:

*Einzahlung:*

$$r_e = r \cdot \left[m + \frac{i \cdot (m-1)}{2}\right] = 1.000 \cdot \left[4 + \frac{0{,}05 \cdot 3}{2}\right] = 4.075$$

$$R_{20} = r_e \cdot \frac{q^n - 1}{q-1} = 4.075 \cdot \frac{1{,}05^{20} - 1}{1{,}05 - 1} = 134.743{,}76$$

*Auszahlung:*

$$r_e^* = r \cdot \left[m + \frac{i \cdot (m+1)}{2}\right] = 1.000 \cdot \left[12 + \frac{0{,}05 \cdot 13}{2}\right] = 12.325$$

$$n = \frac{log\left[\frac{R_n}{r_e^*} \cdot (q-1) + q\right]}{\log(q)} - 1$$

$$n = \frac{log\left[\frac{134.743{,}76}{12.325} \cdot (0{,}05) + 1{,}05\right]}{\log(1{,}05)} - 1 \approx 8{,}5\ Jahre$$

# Lösung:

zu 1) Nachschüssig: $r = R_0 \cdot i = 200.000 * 0{,}07 = 14.000$

Vorschüssig: $r = R_0 \cdot \frac{i}{q} = 200.000 * \frac{0{,}07}{1{,}07} = 13.084{,}11$

zu 2a) Sparbuchmethode: $K_{x,t_0} = K_0 \cdot \left(1 + i \cdot \frac{t_1}{360}\right) \cdot (1+i)^n$

$$K_{31.12} = 80.000 \cdot \left(1 + 0{,}05 \cdot \frac{1}{2}\right) \cdot (1{,}05)^5 = 104.655{,}09$$

zu 2b) $i = \frac{r}{R_0} = \frac{2.500}{104.655{,}09} = 0{,}02388 \approx 2{,}4\%$

zu 2c) $r_e = r \cdot \left[m + \frac{i \cdot (m-1)}{2}\right] = r \cdot \left[2 + \frac{0{,}075 \cdot 1}{2}\right] = r \cdot 2{,}0375$

$$r \cdot 2{,}0375 = R_0 \cdot i = 104.655{,}09 \cdot 0{,}075 = 7.849{,}13$$

$$r = \frac{7.849{,}13}{2{,}0375} = 3.852{,}33$$