

$$1) \left. \begin{array}{l} \begin{vmatrix} 1 & -2 & 1 \\ 2 & x-3 & 3 \\ -1 & 2 & x-3 \end{vmatrix} = \begin{array}{l} x^2 + 6x + 4 \\ \ominus \\ -x - 4x + 6 \end{array} \end{array} \right\} \begin{array}{l} x^2 + 5x + 4 = 0 \\ (x+4)(x+1) = 0 \end{array}$$

$$D = x \in \mathbb{R} \setminus \{-4, -1\}$$

$$x = -3 \Rightarrow \det(A) = (-3)^2 + 5 \cdot (-3) + 4 = -2$$

$$A_{11} = + \begin{vmatrix} -3 & 3 \\ 2 & -3 \end{vmatrix} = 3 \quad A_{12} = - \begin{vmatrix} 2 & 3 \\ -1 & -3 \end{vmatrix} = +3 \quad A_{13} = + \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = 1$$

$$A_{21} = - \begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix} = -4 \quad A_{22} = + \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} = -2 \quad A_{23} = - \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} = 0$$

$$A_{31} = + \begin{vmatrix} -2 & 1 \\ -3 & 3 \end{vmatrix} = -3 \quad A_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1 \quad A_{33} = + \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} = 1$$

$$A^{-1} = -\frac{1}{2} \cdot \begin{pmatrix} 3 & 3 & 1 \\ -4 & -2 & 0 \\ -3 & -1 & 1 \end{pmatrix}^T = -\frac{1}{2} \cdot \begin{pmatrix} 3 & -4 & -3 \\ 3 & -2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\vec{x} = A^{-1} \cdot \vec{b} = -\frac{1}{2} \begin{pmatrix} 3 & -4 & -3 \\ 3 & -2 & -1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$$

$$\vec{x} = -\frac{1}{2} \cdot \begin{pmatrix} 3 & -4 & +9 \\ 3 & -2 & +3 \\ 1 & +0 & -3 \end{pmatrix} = -\frac{1}{2} \cdot \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$$

3) Eigenwerte λ : $|A - \lambda \cdot E| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 2 & 2-\lambda & -2 \\ 3 & 1 & 1-\lambda \end{vmatrix} = \lambda^3 - 5\lambda^2 + 2\lambda + 8$$

$$\lambda = -1$$

$$\begin{array}{r} \lambda^3 - 5\lambda^2 + 2\lambda + 8 \\ - (\lambda^3 + \lambda^2) \\ \hline - 6\lambda^2 + 2\lambda + 8 \\ - (-6\lambda^2 - 6\lambda) \\ \hline 8\lambda + 8 \\ - (8\lambda + 8) \\ \hline - \quad - \end{array}$$

$$\begin{array}{l} (\lambda^2 - 6\lambda + 8) \\ (\lambda - 4)(\lambda - 2) \end{array}$$

$$\lambda_1 = -1$$

$$\lambda_2 = 4$$

$$\lambda_3 = 2$$

$$A = \begin{vmatrix} 1 & -3 & 1 \\ -1 & 2 & -3 \\ 0 & 1 & 4 \end{vmatrix} = \begin{matrix} 8 - 1 + 18 \\ \ominus \\ 4 + 12 - 3 \end{matrix} \begin{matrix} 25 \\ - \\ 13 \end{matrix} \left. \vphantom{\begin{matrix} 1 \\ - \\ 13 \end{matrix}} \right\} \begin{matrix} 12 < 0 \\ \\ \text{regulär} \end{matrix}$$

$$D_1 = \begin{vmatrix} -2 & -3 & 1 \\ -6 & 2 & -3 \\ 16 & 1 & 4 \end{vmatrix} = \begin{matrix} -16 + 144 - 6 \\ \ominus \\ 32 + 72 + 6 \end{matrix} \begin{matrix} 122 \\ \ominus \\ 110 \end{matrix} \left. \vphantom{\begin{matrix} 122 \\ \ominus \\ 110 \end{matrix}} \right\} \begin{matrix} 12 \Rightarrow x_1 = \frac{12}{12} \\ \\ x_1 = 1 \end{matrix}$$

$$D_2 = \begin{vmatrix} 1 & -2 & 1 \\ -1 & -6 & -3 \\ 0 & 16 & 4 \end{vmatrix} = \begin{matrix} -24 + 12 - 16 \\ \ominus \\ -12 + 8 - 48 \end{matrix} \begin{matrix} -28 \\ \ominus \\ -52 \end{matrix} \left. \vphantom{\begin{matrix} -28 \\ \ominus \\ -52 \end{matrix}} \right\} \begin{matrix} 24 \Rightarrow x_2 = \frac{24}{12} \\ \\ x_2 = 2 \end{matrix}$$

$$D_3 = \begin{vmatrix} 1 & -3 & -2 \\ -1 & 2 & -6 \\ 2 & 1 & 16 \end{vmatrix} = \begin{matrix} 30 + 36 + 2 \\ \ominus \\ -8 + 48 - 6 \end{matrix} \begin{matrix} 70 \\ \ominus \\ 34 \end{matrix} \left. \vphantom{\begin{matrix} 70 \\ \ominus \\ 34 \end{matrix}} \right\} \begin{matrix} 36 \Rightarrow x_3 = \frac{36}{12} \\ \\ x_3 = 3 \end{matrix}$$