$$1)\quad \begin{vmatrix} 1 & -2 & 1 \\ 2 & x & 3 \\ -1 & 2 & x \end{vmatrix} = \left.\begin{array}{ccc} x^2 + 6 + 4 & = & x^2 + 10 \\ \ominus & & \ominus \\ -x - 4x + 6 & & -5x + 6 \end{array}\right\} x^2 + 5x + 4$$

$$\mathrm{Det}(A) = x^2 + 5x + 4 = 0$$

$$(x+1)(x+4) = 0 \qquad x_1 = -1 \ \cup\ x_2 = -4$$

$$x^2 + p \cdot x + q = 0 \quad \Rightarrow \quad x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$x_{1/2} = -\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4}$$

$$-\frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}} = -\frac{5}{2} \pm \sqrt{\frac{9}{4}} = \begin{cases} \nearrow -\frac{5}{2} + \frac{3}{2} = -1 \\ \searrow -\frac{5}{2} - \frac{3}{2} = -4 \end{cases}$$

$$\mathbb{L} = x \in \mathbb{R} \setminus \{-4; -1\} \Rightarrow \mathrm{Det}(A) <> 0$$

$\Rightarrow$ regulär

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & -3 & 3 \\ -1 & 2 & -3 \end{pmatrix} \rightarrow Det(A) = -2 \qquad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$$

$$A_{11} = + \begin{vmatrix} -3 & 3 \\ 2 & -3 \end{vmatrix} = 3 \qquad A_{12} = - \begin{vmatrix} 2 & 3 \\ -1 & -3 \end{vmatrix} = +3 \qquad A_{13} = + \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} = 1$$

$$A_{21} = - \begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix} = -4 \qquad A_{22} = + \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} = -2 \qquad A_{23} = - \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} = 0$$

$$A_{31} = + \begin{vmatrix} -2 & 1 \\ -3 & 3 \end{vmatrix} = -3 \qquad A_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1 \qquad A_{33} = + \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} = 1$$

$$A^{-1} = -\frac{1}{2} \cdot \begin{pmatrix} 3 & 3 & 1 \\ -4 & -2 & 0 \\ -3 & -1 & 1 \end{pmatrix}^{T} = -\frac{1}{2} \begin{pmatrix} 3 & -4 & -3 \\ 3 & -2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A \cdot \vec{x} = \vec{b} \quad \Leftrightarrow \quad \vec{x} = A^{-1} \cdot \vec{b}$$

$$\underbrace{-\frac{1}{2} \cdot \begin{pmatrix} 3 & -4 & -3 \\ 3 & -2 & -1 \\ 1 & 0 & 1 \end{pmatrix}}_{A^{-1}} \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}}_{\vec{b}} = -\frac{1}{2} \cdot \begin{pmatrix} 3 - 4 + 9 \\ 3 - 2 + 3 \\ 1 + 0 - 3 \end{pmatrix}$$

$$-\frac{1}{2} \cdot \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix} = \vec{x}$$

3) $|A - \lambda E| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 2 & 2-\lambda & -2 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^3 - 5\lambda^2 + 2\lambda + 8 = 0 \quad \left.\begin{matrix} \lambda_1 = -1 \\ \lambda_2 = 2 \\ \lambda_3 = 4 \end{matrix}\right.$$

$: (\lambda + 1)$

2)
$$\begin{vmatrix} 1 & -3 & 1 \\ -1 & 2 & -3 \\ 2 & 1 & 4 \end{vmatrix} = \begin{array}{c} 8 - 1 + 18 \\ \ominus \\ 4 + 12 - 3 \end{array} \quad \left.\begin{array}{c} 25 \\ - \\ 13 \end{array}\right\} 12$$

$$\begin{vmatrix} -2 & -3 & 1 \\ -6 & 2 & -3 \\ 16 & 1 & 4 \end{vmatrix} = \begin{array}{c} -16 + 144 - 6 \\ \ominus \\ 32 + 72 + 6 \end{array} \quad \left.\begin{array}{c} 122 \\ \ominus \\ 110 \end{array}\right\} \begin{array}{l} 12 \\ x_1 = 1 \end{array}$$

$$\begin{vmatrix} 1 & -2 & 1 \\ -1 & -6 & -3 \\ 2 & 16 & 4 \end{vmatrix} = \begin{array}{c} -24 + 12 - 16 \\ \ominus \\ -12 + 8 - 48 \end{array} \quad \left.\begin{array}{c} -28 \\ \ominus \\ -52 \end{array}\right\} \begin{array}{l} 24 \\ x_2 = 2 \end{array}$$

$$\begin{vmatrix} 1 & -3 & -2 \\ -1 & 2 & -6 \\ 2 & 1 & 16 \end{vmatrix} = \begin{array}{c} 32 + 36 + 2 \\ \ominus \\ -8 + 48 - 6 \end{array} \quad \left.\begin{array}{c} 70 \\ \ominus \\ 34 \end{array}\right\} \begin{array}{l} 36 \\ x_3 = 3 \end{array}$$