I inverse Matrix:

$A \cdot \vec{x} = \vec{b} \quad | \, A^{-1}_{L}$

1) inverse A^{-1} $\qquad \vec{x} = A^{-1} \cdot \vec{b}$

↳ $Det(A) <> 0$: regulär

↳ $A^{-1} = \frac{1}{Det(A)} \cdot \begin{pmatrix} A_{11} & \\ & A_{nn} \end{pmatrix}^T$

2) Vektorprodukt $A^{-1} \cdot \vec{b}$

II Eigenvektor / -werte

1. $\mathrm{Det}\,(A - \lambda \cdot E) = 0$

Eigenwerte

2. $A \cdot \vec{x} = \lambda \cdot \vec{x}$

Eigenvektor

1) a) $\begin{vmatrix} 1 & -2 & 1 \\ 2 & x & 3 \\ -1 & 2 & x \end{vmatrix} = \begin{matrix} x^2 + 6 + 4 \\ - \\ -x - 4x + 6 \end{matrix} \Bigg\} \begin{matrix} x^2 + 5x + 4 = 0 \\ (x+4)(x+1) = 0 \end{matrix}$

$x \in \{-4; -1\}$

$x \in \mathbb{R} \setminus \{-4; -1\}$

b) $x = -3 \Rightarrow Det(A) = (-3)^2 - 15 + 4 = -2$

$$A^{-1} = -\frac{1}{2} \cdot \begin{pmatrix} 3 & -4 & -3 \\ 3 & -2 & -1 \\ 1 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} = \vec{b}$$

$\vec{x} = (-4; -2; 1)^T$

3) $A = \left| \begin{pmatrix} 2-\lambda & 1 & 2 \\ 2 & 2-\lambda & -2 \\ 3 & 1 & 1-\lambda \end{pmatrix} \right| = 0 \qquad \det(A - \lambda \cdot E) = 0$

$$\boxed{\begin{aligned} \vec{x}_{(-1)} &= \tfrac{1}{7} \cdot \begin{pmatrix} -8\beta \\ 10\beta \\ 7\beta \end{pmatrix} \\ \vec{x}_4 &= (\beta; 0; \beta)^T \end{aligned}}$$

$$-\lambda^3 + 5\lambda^2 - 2\lambda - 8 = 0$$

$$-(\lambda+1)(\lambda-2)(\lambda-4) = 0 \qquad \Rightarrow \mathbb{L} = \{1; 2; 4\}$$

Eigenvektor für $\lambda = 2$: $\qquad A \cdot \vec{x} = 2 \cdot \vec{x}$

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \left| \begin{aligned} 2x_1 + x_1 + 2x_3 &= 2x_1 \\ 2x_1 + 2x_2 - 2x_3 &= 2x_1 \\ 3x_1 + x_2 + x_3 &= 2x_3 \end{aligned} \right| \begin{aligned} &\mid -2x_1 \\ &\mid -2x_2 \\ & \end{aligned}$$

$x_3 = \beta$

$\vec{x} = \begin{pmatrix} \beta \\ -2\beta \\ \beta \end{pmatrix}$

$$\begin{aligned} x_2 + 2x_3 &= 0 \\ 2x_1 - 2x_3 &= 0 \end{aligned}$$

$$\left. \begin{aligned} x_2 &= -2x_3 \\ x_1 &= x_3 \end{aligned} \right\} \quad \begin{aligned} 3 \cdot x_3 - 2x_3 + x_3 &= 2x_3 \\ 0 &= 0 \end{aligned}$$