$$A \cdot \vec{x} = (-1) \cdot \vec{x}$$

$$\left| \begin{array}{l} -2x_1 + x_2 + x_3 = -x_1 \\ x_2 = -x_2 \\ -2x_2 - x_3 = -x_3 \end{array} \right| \Leftrightarrow \left| \begin{array}{l} -x_1 + x_2 + x_3 = 0 \\ 2x_2 = 0 \\ -2x_2 = 0 \end{array} \right| \left. \begin{array}{l} \\ \\ \end{array} \right\} x_2 = 0$$

$$-x_1 + 0 + x_3 = 0 \qquad r = x_1$$

$$-r + x_3 = 0$$

$$x_3 = r$$

$$\Rightarrow \text{Eigenvektor: } \begin{pmatrix} r \\ 0 \\ r \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -3 & 0 \end{pmatrix}$$

$$\det[A - \lambda \cdot E] = 0$$

$$\det \begin{bmatrix} 1-\lambda & 0 & 0 \\ -1 & 3-\lambda & 0 \\ 0 & -3 & -\lambda \end{bmatrix} = (1-\lambda)\cdot(3-\lambda)\cdot(-\lambda) = 0$$

Eigenwerte: $\lambda_1 = 1$; $\lambda_2 = 3$; $\lambda_3 = 0$

$\underline{\lambda = 3}$

$$A \cdot \vec{x} = 3 \cdot \vec{x}$$

$$\left| \begin{array}{rl} x_1 & = 3x_1 \\ -x_1 + 3x_2 & = 3x_2 \\ -3x_2 & = 3x_3 \end{array} \right| \Leftrightarrow \left| \begin{array}{rl} -2x_1 & = 0 \\ -x_1 & = 0 \\ -x_2 & = x_3 \end{array} \right|$$

$x_2 = r$

$$\begin{pmatrix} 0 \\ r \\ -r \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & x & 3 \\ -1 & 2 & x \end{pmatrix} \Rightarrow Det(A) = x^2 + 5x + 4$$

$$= (x+4)\cdot(x+1) = 0$$

$$\Rightarrow x_1 = -4 \vee x_2 = -1$$

LL: $x \in \mathbb{R} \setminus \{-4; -1\}$, da somit A regulär ist.

$$x = -3 \quad \Rightarrow \quad Det(A) = (-3+4)\cdot(-3+1) = -2$$

$$A^{-1} = \frac{1}{2}\cdot \begin{pmatrix} -3 & 4 & 3 \\ -3 & 2 & 1 \\ -1 & 0 & -1 \end{pmatrix} \quad \Rightarrow \quad \vec{x} = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & -2 & 0 & 2 \\ -3 & 2 & 2 & 3 \end{array}\right) \quad I \cdot (-2) \quad I \cdot 3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 5 & 6 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 3 & 6 \end{array}\right) \begin{array}{lll} \rightarrow & x_1 + 2 = 1 & x_1 = -1 \\ \rightarrow & -2x_2 - 4 = 0 & x_2 = -2 \\ \rightarrow & x_3 = 2 & \end{array}$$

$$\vec{x} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ -1 & -1 & -2 & 1 \\ 3 & 1 & \alpha & \beta \end{array}\right) \quad \cdot(-3)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & -5 & \alpha-9 & \beta-6 \end{array}\right) \quad \cdot 5$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & \alpha-4 & \beta+9 \end{array}\right)$$

Zahl | egal : $\alpha <> 4$: $Det(A) = 1 \cdot 1 \cdot (\alpha - 4) <> 0$

$\Downarrow$

eindeutige Lösung

$Rang(A) = $ Maximalrang $= 3 = Rang(A|b)$

Null | Zahl : $\qquad \alpha = 4 \wedge \beta <> -9$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & \beta - 9 \end{array}\right)$$

$$\mathrm{Det}(A) = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \quad \Rightarrow \mathrm{Rang}(A) = 2$$

$$\mathrm{Det}(A|b) = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & \beta - 9 \end{vmatrix} = \beta - 9 <> 0 \quad \Rightarrow \mathrm{Rang}(A|b) = 3$$

$\mathrm{Rang}(A) <> \mathrm{Rang}(A|b) \Rightarrow$ keine Lösung

Null | Null $\qquad \alpha = 4 \wedge \beta = -9$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\mathrm{Det}(A) = 1 \Rightarrow \mathrm{Rang}(A) = 2 = \mathrm{Rang}(A|b)$

$3 - 2$ Variable frei wählbar: $\boxed{x_3 = r}$

$$\vec{x} = \begin{pmatrix} -4 + r \\ 3 - r \\ r \end{pmatrix}$$

$$x_1 + 2x_2 + r = 2$$

$$x_2 + r = 3 \quad \Rightarrow \quad \boxed{x_2 = \underline{3 - r}}$$

$$\vec{x} = \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} + r \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$x_1 + 2 \cdot (3 - r) + r = 2$$

$$x_1 + 6 - 2r + r = 2$$

$$\boxed{x_1 = -4 + r}$$