S 51 Nr. 3 $\quad x \cdot \vec{v_1}_\alpha + y \cdot \vec{v_2} + z \cdot \vec{v_3}_\beta = \vec{0}$

$$\left| \begin{array}{rrrl} x & +2y & -3z & = 0 \\ \alpha x & + y & -\beta \cdot z & = 0 \\ -x & & +z & = 0 \end{array} \right| \Rightarrow \boxed{x = z}$$

$$\left| \begin{array}{l} 2y + z - 3z = 2y - 2z = 0 \\ y + \alpha \cdot z - \beta \cdot z = y + (\alpha - \beta) \cdot z = 0 \end{array} \right| \; y = z$$

$$z + (\alpha - \beta) \cdot z = 0$$

$$\underbrace{(1 + \alpha - \beta)}_{\neq 0} \cdot z = 0$$

$$\alpha = \beta - 1$$

Antwort: $\alpha \neq \beta - 1$, somit lineare Unabhängigkeit

$$\left|\begin{array}{l} x + 2y - 3z = 0 \\ \alpha \cdot x + 1y - \beta \cdot z = 0 \\ -x + 0 + z = 0 \end{array}\right| \begin{array}{l} \cdot(-\alpha) \\ + \\ + \end{array}$$

$-2\alpha y + y$

$(1-2\alpha)y$

$$\left|\begin{array}{l} x + 2y - 3z = 0 \\ 0 \quad (1-2\alpha)y + (3\alpha - \beta)\cdot z = 0 \\ 0 \quad +2y - 2\cdot z = 0 \end{array}\right| \begin{array}{l} \\ \cdot(-2) \\ (1-2\alpha) \end{array}$$

$$\left|\begin{array}{l} x + 2y - 3z = 0 \\ 0 \quad -(2-4\alpha)y + (2\beta - 6\alpha)\cdot z = 0 \\ 0 \quad +(2-4\alpha)\cdot y + (4\alpha - 2)\cdot z = 0 \end{array}\right| \begin{array}{l} \\ \\ + \end{array}$$

$2\beta - 2\alpha - 2 = 0$

$\alpha = \beta - 1$

$$\left|\begin{array}{l} x + 2y - 3z = 0 \\ 0 \quad -(2-4\alpha)y + (2\beta - 6\alpha) z = 0 \\ 0 \quad 0 + (2\beta - 2\alpha - 2)\cdot z = 0 \end{array}\right|$$

1) $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$; $\vec{b} \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$; $\vec{c} = \begin{pmatrix} 6 \\ -5 \\ -1 \end{pmatrix}$

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + p \begin{pmatrix} 4 - 2 \\ -2 - 1 \\ 1 - 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + p \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} = \vec{c}$$

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + p \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \\ -1 \end{pmatrix} \quad | - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$p \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix} \qquad \left.\begin{aligned} 2 \cdot p &= 2 \\ -3 \cdot p &= -6 \\ -2p &= -4 \end{aligned}\right\} \; p = 2 \; ✓$$

2) $g_1: \vec{x}_1 = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + p \cdot \begin{pmatrix} 4-3 \\ -2-2 \\ 2+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + p \cdot \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$

$\cdot(2)$

$g_2: \vec{x}_2 = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + r \cdot \begin{pmatrix} -1+3 \\ -4-4 \\ 8-2 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + r \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix}$

Richtungsvektoren sind linear abhängig
=> Punktprobe :

$$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + r \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix} \quad \Big| - \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -2 \\ -3 \end{pmatrix} = r \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix} \quad \begin{matrix} \to r = 3 \\ \to r = -\tfrac{1}{4} \end{matrix} \quad ↯ \qquad \underline{\text{parallel}}$$

EBENE

1) $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\vec{b} \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$; $\vec{c} = \begin{pmatrix} 6 \\ -5 \\ -1 \end{pmatrix}$

$$e: \vec{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + p \cdot \begin{pmatrix} 4-2 \\ -2-1 \\ 1-3 \end{pmatrix} + \gamma \cdot \begin{pmatrix} 6-2 \\ -5-1 \\ -1-3 \end{pmatrix}$$

$$e \quad \vec{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \underbrace{p \cdot \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} + \gamma \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix}}_{\text{linear abhängig}}$$

$$\Rightarrow g: \vec{x} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$$

S77 Nr. 1) b)

$$e: \vec{x} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} + p \begin{pmatrix} -3-0 \\ -2-2 \\ 2-1 \end{pmatrix} + r \begin{pmatrix} 9-0 \\ 2+2 \\ -4-1 \end{pmatrix}$$

$$\vec{x} = \boxed{\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}} + p \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + r \begin{pmatrix} 9 \\ 4 \\ -5 \end{pmatrix}$$

$$\vec{n}: \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 9 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 0-4 \\ 9-15 \\ -12-0 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ -12 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$$

$ax + by + cz = d$

$2x + 3y + 6z = d$

$2 \cdot 0 + 3 \cdot (-2) + 6 \cdot 1 = 0$

$2x + 3y + 6z = 0$

S 82 Nr. 2) $\qquad e: 2x - 3y + 4z = 5$

$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}: \quad 2 \cdot 1 - 3 \cdot (-1) + 4 \cdot 0 = 5 \quad \checkmark$

$\begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix}: \quad 2 \cdot 0 - 3 \cdot (-3) + 4(-1) = 5 \quad \checkmark$

$\begin{pmatrix} 1 \\ 0 \\ 3/4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}: \quad 2 \cdot 2 - 3 \cdot 1 + 4 \cdot 1 = 5 \quad \checkmark$

$$e: \vec{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + p \cdot \begin{pmatrix} 0 - 1 \\ -3 + 1 \\ -1 - 0 \end{pmatrix} + r \begin{pmatrix} 2 - 1 \\ 1 + 1 \\ 1 - 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 - 1 \\ 0 + 1 \\ 3/4 - 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + p \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} + r \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 3/4 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$$

5)

$$e: \vec{x} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-4 \\ -6-2 \\ -4-3 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix}$$

$$\left.\begin{matrix} 3x + 8y + 7z = d \\ 3 - 16 + 7 = -6 \end{matrix}\right\} \boxed{3x + 8y + 7z = -6}$$

e_2: $2x - 3y + 4z = 5$

$$\begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix} \cdot \varepsilon = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \quad ⚡$$

$$\begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix} \times \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 32 + 21 \\ 14 - 12 \\ -9 - 16 \end{pmatrix} = \begin{pmatrix} 53 \\ 2 \\ -25 \end{pmatrix}$$

$$\left| \begin{array}{l} 2x - 3y + 4z = 5 \\ 3x + 8y + 7z = -6 \end{array} \right| \qquad z = 0$$

$$\left| \begin{array}{l} 2x - 3y = 5 \\ 3x + 8y = -6 \end{array} \right| \quad \begin{array}{l} \cdot (-3) \\ \cdot 2 \end{array}$$

$\underline{V} \mathrel{\widehat{=}}$ FÜNF

$$\left(\begin{array}{l} -6x + 9y = -15 \\ 6x + 16y = -12 \end{array} \right)$$

$$25y = -27$$

$$y = -27/25$$

$$x = \ldots$$