1) $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$

$$\left|\begin{array}{rrrcl} \beta & +3\alpha & -3\gamma & = & 0 \\ -2\beta & -\alpha & +\gamma & = & 0 \\ \beta & +2\alpha & +4\gamma & = & 0 \end{array}\right| \quad \cdot 2 \quad \cdot(-1) \quad \text{Pivot}$$

$$\left|\begin{array}{rrrcl} \beta & +3\alpha & -\gamma & = & 0 \\ 0 & 5\alpha & -5\gamma & = & 0 \\ 0 & -\alpha & +7\gamma & = & 0 \end{array}\right| \quad \cdot \tfrac{1}{5} \qquad \text{Pivot}$$

$$\left|\begin{array}{rrrcl} \beta & +3\alpha & -\gamma & = & 0 \\ 0 & \alpha & -\gamma & = & 0 \\ 0 & 0 & 6\gamma & = & 0 \end{array}\right| \quad \begin{array}{l} \beta = 0 \\ \alpha = 0 \\ \to \gamma = 0 \end{array}$$

$\alpha = \beta = \gamma = 0$ Triviallösung -> Basis -> Dimension 3

$$\alpha \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{d}$$

$$\left|\begin{array}{rrrcr} 3\alpha & +\beta & -3\gamma & = & 0 \\ -1\alpha & -2\beta & +\gamma & = & 5 \\ 2\alpha & +\beta & +4\gamma & = & 5 \end{array}\right| \quad I\cdot 2,\ I\cdot 3 \qquad \text{Pivot}$$

$$\left|\begin{array}{rrrcr} -\alpha & -2\beta & +\gamma & = & 5 \\ 0 & -5\beta & & = & 15 \\ 0 & -3\beta & +6\gamma & = & 15 \end{array}\right| \qquad \begin{array}{l} \alpha = 2 \\ \beta = -3 \\ \Rightarrow 6\gamma = 6 \Rightarrow \gamma = 1 \end{array}$$

$$\Rightarrow \quad 2 \cdot \vec{a} - 3\vec{b} + \vec{c} = \vec{d}$$

2)

$$\left(\begin{array}{cccc|c} -1 & 1 & -2 & -7 & 0 \\ 1 & 1 & -1 & -2 & 0 \\ -2 & 0 & 3 & -1 & 0 \\ 2 & 2 & 3 & 1 & 0 \end{array}\right) \quad + \quad |\cdot(-2) \;+ \quad |\cdot 2 \;+$$

$$\left(\begin{array}{cccc|c} -1 & 1 & -2 & -7 & 0 \\ 0 & 2 & -3 & -9 & 0 \\ 0 & -2 & 7 & 13 & 0 \\ 0 & 4 & -1 & -13 & 0 \end{array}\right) \quad + \quad |\cdot(-2) \;+$$

$$\left(\begin{array}{cccc|c} -1 & 1 & -2 & -7 & 0 \\ 0 & 2 & -3 & -9 & 0 \\ 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 5 & 5 & 0 \end{array}\right) \quad \begin{array}{l} |\cdot(-\frac{1}{4}) \\ |\cdot\frac{1}{5} \end{array} \;+$$

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \end{array}$$

5) a) Variante I : $\Rightarrow$ lineare Abhängigkeit

$$\left|\begin{array}{rrrl} \alpha & +\beta & +\gamma & =0 \\ 2\alpha & -4\beta & +8\gamma & =0 \\ -2\alpha & +3\beta & -7\gamma & =0 \end{array}\right| \quad \begin{array}{l} \cdot(-2) \quad \cdot 2 \\ + \\ - \end{array}$$

$$\left|\begin{array}{rrrl} \alpha & +\beta & +\gamma & =0 \\ 0 & -6\beta & +6\gamma & =0 \\ 0 & 5\beta & -5\gamma & =0 \end{array}\right| \quad \begin{array}{l} \cdot \frac{1}{6} \\ \cdot \frac{1}{5} \quad + \end{array}$$

$$\left|\begin{array}{rrrl} \alpha & +\beta & +\gamma & =0 \\ 0 & -\beta & +\gamma & =0 \\ 0 & 0 & 0 & =0 \end{array}\right|$$

Setze $\beta = 1 \Rightarrow \gamma = 1 \Rightarrow \alpha = -2$

$\Rightarrow$ linear abhängig

Variante $\underline{II}$: Punktprobe

$$g_{AB}: \quad \vec{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \varepsilon \begin{pmatrix} 1 - 1 \\ -4 - 2 \\ 3 + 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$$

$$\vec{c} = \begin{pmatrix} 1 \\ 8 \\ -7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix} \quad \Big| - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 6 \\ -5 \end{pmatrix} = \varepsilon \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix} \quad \left.\begin{array}{l} \rightarrow \varepsilon = \mathbb{R} \\ \rightarrow \varepsilon = -1 \\ \rightarrow \varepsilon = -1 \end{array}\right\} \varepsilon = -1$$

Punkt C liegt auf der Geraden durch AB.

1) a) $\vec{x}_1 = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 8 \\ -4 \end{pmatrix}$, $\vec{x}_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$

1. lineare Abhängigkeit der Richtungsvektoren: $\begin{pmatrix} -2 \\ 8 \\ -4 \end{pmatrix} = \gamma \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ $\quad \gamma = -2$, $\gamma = -2$, $\gamma = -2$

$\Rightarrow$ gleiche Steigung

2. Punktprobe:

$$\begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \quad \Big| - \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -8 \\ 3 \end{pmatrix} = \beta \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \qquad \begin{matrix} \beta = 3 \\ \beta = 2 \end{matrix} \quad \lightning$$

$\Longrightarrow$ parallel

5) $\vec{x}_1 = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \alpha \underline{\underline{\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}}}$; $\vec{x}_2 = \begin{pmatrix} 2 \\ -6 \\ -3 \end{pmatrix} + \beta \underline{\underline{\begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix}}}$

1. lineare Abhängigkeit: $\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \beta \begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix}$ $\quad \beta = -\frac{1}{2}$, $\beta = -\frac{2}{5}$ ↯

$\Rightarrow$ nicht lineare Abhängig

2. $g_1 = g_2$ $\quad \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \\ -3 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ -5 \\ -8 \end{pmatrix}$

$$\left| \begin{array}{rrr} -\alpha & -2\beta & = 1 \\ 2\alpha & +5\beta & = -3 \\ 5\alpha & +8\beta & = -8 \end{array} \right| \begin{array}{l} \cdot 2 \quad \cdot 5 \\ + \\ + \end{array} \left| \begin{array}{rrr} -\alpha & -2\beta & = 1 \\ 0 & \beta & = -1 \\ 0 & -2\beta & = -3 \end{array} \right|$$

$\beta = -1 \wedge \beta = \frac{3}{2}$ ↯ $\Rightarrow$ windschief