$$\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$$ Linearkombination

$$\alpha \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 5 \\ -1 \\ 0 \end{pmatrix} + \gamma \cdot \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left| \begin{array}{lllll} 2\alpha & + 5\beta & - 2\gamma & = & 0 \\ \alpha & - \beta & + 2\gamma & = & 0 \\ 3\alpha & & + 3\gamma & = & 0 \end{array} \right| \Leftrightarrow \alpha = -\gamma$$

$$\left| \begin{array}{l} -2\gamma + 5\beta - 2\gamma = +5\beta - 4\gamma = 0 \\ -\gamma - \beta + 2\gamma = -\beta + \gamma = 0 \end{array} \right| \Leftrightarrow \beta = \gamma$$

$$5\gamma - 4\gamma = \gamma = 0$$

$$\alpha = \beta = \gamma = 0$$ Triviallösung

$\Rightarrow$ Basis der Dimension 3

$$\text{Pivot}\left|\begin{array}{l} 2\alpha + 5\beta - 2\gamma = 6 \\ \alpha - \beta + 2\gamma = 5 \\ 3\alpha + 3\gamma = -6 \end{array}\right| \begin{array}{l} \\ \cdot(-2) \quad \cdot(-3) \\ \\ \end{array}$$

$\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{d}$

$$\left|\begin{array}{l} \alpha - \beta + 2\gamma = 6 \\ 0 \quad 7\beta - 6\gamma = -4 \\ 0 \quad +3\beta - 3\gamma = -21 \end{array}\right| \begin{array}{l} \\ \\ :3 \end{array}$$

$\beta - \gamma = -7$

Pivot $\cdot(-7)$

$$\left|\begin{array}{l} \alpha - \beta + 2\gamma = 6 \\ 0 \quad \beta - \gamma = -7 \\ 0 \quad 0 \quad \gamma = +45 \end{array}\right|$$

$\alpha - 38 + 90 = 6$ $\quad \alpha = -48$

$\beta - 45 = -7$ $\quad \beta = 38$

$$\alpha \cdot \begin{pmatrix} 1 \\ x \\ -1 \end{pmatrix} + \beta \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \gamma \cdot \begin{pmatrix} -3 \\ -y \\ 1 \end{pmatrix} = \vec{0}$$

$$\left| \begin{array}{rcrcrl} \alpha & + & 2\beta & - & 3\gamma & = 0 \\ x \cdot \alpha & + & \beta & - & y \cdot \gamma & = 0 \\ -\alpha & & & + & \gamma & = 0 \end{array} \right| \Rightarrow \quad \alpha = \gamma$$

$$\left| \begin{array}{l} 2\beta - 2\gamma = 0 \\ \beta + (x-y) \cdot \gamma = 0 \end{array} \right| \qquad \beta = \gamma$$

$$\gamma + (x-y) \cdot \gamma = 0$$

$$\underbrace{\underbrace{(1 + x - y)}_{0} \cdot \gamma = 0}_{0 = 0}$$

Antwort:
linear unabhängig
$x - y <> (-1)$