$(\mathbb{R}^3, +, *, \mathbb{R})$ $\quad 5\cdot\begin{pmatrix}2\\3\\4\end{pmatrix} - \frac{1}{2}\cdot\begin{pmatrix}1\\2\\5\end{pmatrix}$

$(\mathbb{R}, +, *)$ -> Körper

$(\mathbb{R}^3, +)$ -> abelsche Gruppe

binär : $\left.\vec{a} + \vec{b} \to \vec{c}\ \right\} \in \mathbb{R}^3$

neutral : $\vec{a} + \vec{0} = \vec{a}$

invers : $\vec{a} + (-\vec{a}) = \vec{0}$ (Aktion = Reaktion)

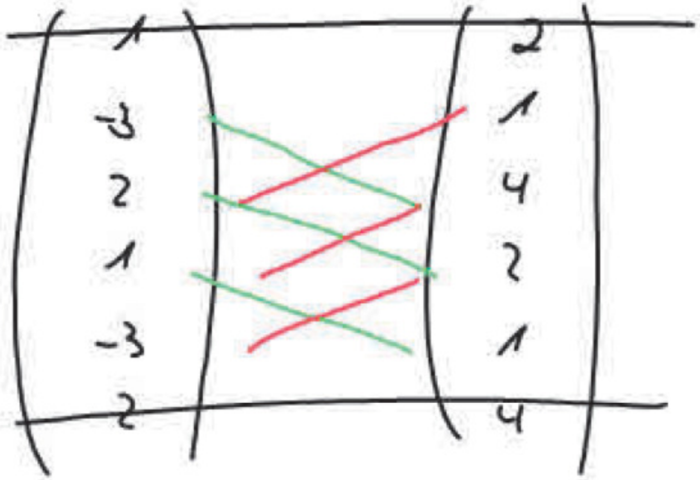
asso. } komponentenweise
kom. } Addition

$$\begin{pmatrix}1\\2\\-3\end{pmatrix} + \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}1+3\\2+1\\-3+2\end{pmatrix} = \begin{pmatrix}4\\3\\-1\end{pmatrix}$$

$\mathbb{R} \times \mathbb{R}^3 - \mathbb{R}^3$

- $\alpha \cdot \vec{a} \to \vec{a} \qquad | \ \mathbb{R} \cdot \mathbb{R}^3 \to \mathbb{R}^3$ Skalar-
- $\beta \cdot (\gamma \cdot \vec{a}) = (\beta \cdot \gamma) \cdot \vec{a}$ assoz.
- $1 \cdot \vec{a} = \vec{a} \qquad 1 \in \mathbb{R}$ neutral
- $(\alpha + \beta) \cdot \vec{a} = \alpha \cdot \vec{a} + \beta \cdot \vec{a}$ distributiv

$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} :$$



$$\underbrace{\begin{pmatrix} -3 \cdot 4 - 1 \cdot 2 \\ 2 \cdot 2 - 4 \cdot 1 \\ 1 \cdot 1 - 2 \cdot (-3) \end{pmatrix}}$$

$$\begin{pmatrix} -14 \\ 0 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -10 - 4 \\ 8 - 15 \\ 3 + 4 \end{pmatrix} = \begin{pmatrix} -14 \\ -7 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 + 12 \\ -4 + 4 \\ 6 + 1 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \\ 7 \end{pmatrix}$$

$\vec{a}\ ;\ \vec{b}\ ;\ \vec{c}$

$$\alpha \cdot \vec{a} + \beta \cdot \vec{b} = \gamma \cdot \vec{c} \qquad \Big| -\gamma\vec{c}$$

$$\underbrace{\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c}}_{\text{Linearkombination}} = \vec{0}$$

$\alpha = \beta = \gamma = 0 \qquad$ (Triviallösung)