$a + b \cdot \sqrt{2}\,;\quad a; b \in \mathbb{Q}$ $\qquad *$ Multiplikation

binär:

$$
\left.
\begin{aligned}
&(a_1 + b_1\sqrt{2}) * (a_2 + b_2\sqrt{2})\\
&a_1 \cdot a_2 + a_2 b_1 \sqrt{2} + a_1 b_2 \sqrt{2} + b_1 b_2 \cdot 2\\
&\underbrace{(a_1 \cdot a_2 + 2 \cdot b_1 b_2)}_{\substack{\mathbb{Q}\\ \downarrow \\ a_3}} + \underbrace{(a_2 b_1 + a_1 b_2)}_{\substack{\mathbb{Q}\\ \downarrow \\ b_3}} \cdot \sqrt{2}
\end{aligned}
\right\}\ a_i; b_i \in \mathbb{Q}
$$

$$a_3 + b_3 \cdot \sqrt{2}$$

asso:

$$[(a_1 + b_1\sqrt{2})\cdot(a_2 + b_2\sqrt{2})]\cdot(a_3 + b_3\sqrt{2})$$

$$(a_1a_2 + a_2b_1\sqrt{2} + a_1b_2\sqrt{2} + b_1b_2\cdot 2)\cdot(a_3 + b_3\sqrt{2})$$

$$\left\{\begin{array}{l} a_1a_2a_3 + a_2a_3b_1\sqrt{2} + a_1a_3b_2\sqrt{2} + a_3b_1b_2\cdot 2 \; + \\ a_1a_2b_3\sqrt{2} + a_2b_1b_3\cdot 2 + a_1b_2b_3\cdot 2 + b_1b_2b_3\cdot 2\sqrt{2} \end{array}\right.$$

$\ominus$ $\Rightarrow \partial$

$$(a_1 + b_1\sqrt{2})\cdot[(a_2 + b_2\sqrt{2})\cdot(a_3 + b_3\sqrt{2})]$$

$$\left\{ \quad \ldots \right.$$

Komm:

$$(a_1 + b_1\sqrt{2})(a_2 + b_2\sqrt{2}) = (a_2 + b_2\sqrt{2})\cdot(a_1 + b_1\sqrt{2})$$

neutral:

$$(a+b\sqrt{2})\cdot(\overset{1_a}{\overset{\downarrow}{x}}+\overset{1_b}{\overset{\downarrow}{y}}\sqrt{2}) = a+b\sqrt{2}$$

$$ax + bx\sqrt{2} + ay\sqrt{2} + 2by = a + b\sqrt{2}$$

$$(ax+2by) + (bx+ay)\cdot\sqrt{2} = a + b\sqrt{2}$$

$$\left|\begin{array}{l} ax + 2by = a \\ bx + ay = b \end{array}\right| \rightarrow x = \frac{b-ay}{b}$$

$$a\cdot\frac{b-ay}{b} + 2by = a \qquad | \; T$$

$$\frac{ab-a^2y}{b} + \frac{2b^2y}{b} = \frac{ab}{b} \qquad \Big| -\frac{ab}{b}$$

$$\frac{ab-a^2y+2b^2y-ab}{b} = 0$$

$$a \cdot \frac{b-ay}{b} + 2by = a \qquad | \cdot HN \quad \cdot b$$

$$ab - a^2y + 2b^2y = ab \qquad | -ab \qquad | T_y$$

$$y \cdot (\underbrace{2b^2 - a^2}) = 0$$

$$y = 0 \qquad 2b^2 - a^2 = 0 \to a^2 = 2b^2 \to a = b \cdot \sqrt{2} \quad ↯$$

$$x = \frac{b-ay}{b} = \frac{b - a \cdot 0}{b} = \frac{b}{b} = 1$$

$$1 = 1 + 0 \cdot \sqrt{2} \qquad (a + b \cdot \sqrt{2})(1 + 0 \cdot \sqrt{2})$$

invers: $(a + b\sqrt{2})(x + y\sqrt{2}) = 1 + 0 \cdot \sqrt{2}$

$$x = \frac{a}{a^2 - 2b^2}, \quad y = \frac{b}{2b^2 - a^2}$$