$$\left(\left\{\sqrt{3}\cdot x - y \;;\; x,y\in\mathbb{Q}\right\};\ +,*\right)$$

$$\left(\sqrt{3}\cdot 5 - \tfrac{7}{2}\right) + \left(\sqrt{3}\cdot\left(-\tfrac{2}{3}\right) - \left(-\tfrac{4}{7}\right)\right)$$

$$\sqrt{3}\cdot\underbrace{\left(5-\tfrac{2}{3}\right)}_{\mathbb{Q}} + \underbrace{\left(-\tfrac{7}{2}+\tfrac{4}{7}\right)}_{\mathbb{Q}}$$

[illegible] $:$ $\quad \left(\sqrt{3}\,x_1 - y_1\right) + \left(\sqrt{3}\,x_2 - y_2\right)$

$$\left.\begin{array}{l} \sqrt{3}\,\underbrace{(x_1+x_2)} - \underbrace{(y_1+y_2)} \\ \sqrt{3}\quad x_3 \quad - \quad y_3 \end{array}\right\}\ x_i\,,\ y_i \in \mathbb{Q}$$

assoziativ: $(a_1 + a_2) + a_3 = a_1 + (a_2 + a_3)$

$$\left[(\sqrt{3}\, x_1 - y_1) + (\sqrt{3}\, x_2 - y_2) \right] + (\sqrt{3}\, x_3 - y_3)$$

$$\left(\sqrt{3}\,(x_1 + x_2) - (y_1 + y_2) \right) + (\sqrt{3}\, x_3 - y_3)$$

$$\sqrt{3}\,(x_1 + x_2 + x_3) - (y_1 + y_2 + y_3)$$

$$(\sqrt{3}\, x_1 - y_1) + \left[(\sqrt{3}\, x_2 - y_2) + (\sqrt{3}\, x_3 - y_3) \right]$$

$$(\sqrt{3}\, x_1 - y_1) + \left(\sqrt{3}\,(x_2 + x_3) - (y_2 + y_3) \right)$$

$$\sqrt{3}\,(x_1 + x_2 + x_3) - (y_1 + y_2 + y_3)$$

=

Kommutativ: $a_1 + a_2 = a_2 + a_1$

$$(\sqrt{3}x_1 - y_1) + (\sqrt{3}x_2 - y_2) = (\sqrt{3}x_2 - y_2) + (\sqrt{3}x_1 - y_1)$$

$$\sqrt{3}(x_1 + x_2) - (y_1 + y_2) = \sqrt{3}(x_2 + x_1) - (y_2 + y_1)$$

gilt, da Standardaddition

neutral: $a + \mathbb{1} = a$

$$(\underline{\sqrt{3}\cdot x}\ \underline{-y}) + (\underline{\sqrt{3}\cdot x_1}\ \underline{-y_1}) = \underline{\sqrt{3}\cdot x}\ \underline{-y}$$

$$\sqrt{3}\cdot x + \sqrt{3}\cdot x_1 = \sqrt{3}\cdot(x + x_1) = \sqrt{3}\cdot x \Rightarrow x_1 = 0$$

$$-y - y_1 = -y \quad \Rightarrow y_1 = 0$$

$$\mathbb{1} = \sqrt{3}\cdot 0 - 0$$

inverses: $\quad a + \bar{a} = 1$

$$(\sqrt{3} \cdot x - y) + (\sqrt{3}\,\bar{x} - \bar{y}) = \sqrt{3} \cdot 0 - 0$$

$$\sqrt{3} \cdot (x + \bar{x}) = \sqrt{3} \cdot 0 \qquad \bar{x} = -x$$

$$-y - \bar{y} = 0 \qquad \bar{y} = -y$$

$$\left.\begin{array}{l} \bar{x} = -x \\ \bar{y} = -y \end{array}\right\} \in \mathbb{Q}$$

$$\bar{a} = \sqrt{3} \cdot (-x) - (-y)$$

=> abelsche Gruppe

$*$: assoziativ + kommutativ gilt.

<u>Sinär:</u> $(\sqrt{3}x_1 - y_1) \cdot (\sqrt{3}x_2 - y_2) = \sqrt{3}x_3 - y_3$

gilt für alle $x_i, y_i \in \mathbb{Q}$

$$3x_1x_2 - \sqrt{3}x_1y_2 - \sqrt{3}x_2y_1 + y_1y_2$$

$$\sqrt{3} \cdot \underbrace{(-x_1y_2 - x_2y_1)}_{x_3} - \underbrace{(-3x_1x_2 - y_1y_2)}_{y_3}$$

<u>neutral:</u> $a \cdot 1 = a$

$$(\sqrt{3} \cdot x - y) \cdot \underbrace{(\sqrt{3}x_1 - y_1)}_{1} = \sqrt{3} \cdot x - y$$

$$1 = \sqrt{3} \cdot 0 - (-1)$$

invers: $a \cdot \bar{a} = 1$

$$(\sqrt{3}x - y) \cdot (\sqrt{3}\bar{x} - \bar{y}) = \sqrt{3} \cdot 0 - (-1)$$

$$3x\bar{x} - \sqrt{3}\bar{x}y - \sqrt{3}x\bar{y} + y\bar{y} = \sqrt{3} \cdot 0 + 1$$

$$\sqrt{3}(-\bar{x}y - x\bar{y}) + (3x\bar{x} + y\bar{y}) = \sqrt{3}\,0 + 1$$

$$-\bar{x}y - x\bar{y} = 0 \quad \wedge \quad 3x\bar{x} + y\bar{y} = 1$$

$$\bar{x} = -\frac{x\bar{y}}{y}$$

$$\frac{-3x^2\bar{y}}{y} + \frac{y^2\bar{y}}{y} = 1$$

$$\bar{y}\,\frac{y^2 - 3x^2}{y} = 1$$

$$\boxed{\begin{aligned} \bar{y} &= \frac{y}{y^2 - 3x^2} \\ \bar{x} &= -\frac{x}{y^2 - 3x^2} \end{aligned}}$$