S 16 Nr. 2) $A = \{1; -1\}$: $(A, *)$

Da Standardmultiplikation gilt:

- -> Assoziativgesetz
- -> Kommutativgesetz
- -> $e = 1 \in A$

Cayley - Tabelle

$*$	-1	1
-1	1	-1
1	-1	1

binär

invers:

$$\left.\begin{array}{l} 1 * \overline{1} = 1 \longrightarrow 1 \\ (-1) * \overline{(-1)} = 1 \longrightarrow -1 \end{array}\right\} \overline{x} = x$$

=> abelsche Gruppe

S 25) Nr. 2 $\quad M = \{ a + b \cdot \sqrt{2} \, ; \, a, b \in \mathbb{Q} \}$

$$0 + 1 \cdot \sqrt{2} \in M$$

<u>Siai</u> :

$$\left.\begin{aligned} &(a_1 + b_1 \sqrt{2}) + (a_2 + b_2 \sqrt{2}) \\ &\underbrace{(a_1 + a_2)}_{\mathbb{Q}} + \underbrace{(b_1 + b_2)}_{\mathbb{Q}} \cdot \sqrt{2} \\ &a_3 + b_3 \cdot \sqrt{2} \end{aligned}\right\} a_i ; b_i \in \mathbb{Q}$$

assoziativ

$$\left[(a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2})\right] + (a_3 + b_3\sqrt{2})$$

$$\left[(a_1 + a_2) + (b_1 + b_2)\sqrt{2}\right] + (a_3 + b_3\sqrt{2})$$

$$(a_1 + a_2 + a_3) + (b_1 + b_2 + b_3)\sqrt{2}$$

$$(a_1 + b_1\sqrt{2}) + \left[(a_2 + b_2\sqrt{2}) + (a_3 + b_3\sqrt{2})\right]$$

$$(a_1 + b_1\sqrt{2}) + \left[(a_2 + a_3) + (b_2 + b_3)\sqrt{2}\right]$$

kommutativ

$$(a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2})$$

$$(a_1 + a_2) + (b_1 + b_2)\sqrt{2}$$

$$(a_2 + b_2\sqrt{2}) + (a_1 + b_1\sqrt{2})$$

$$(a_2 + a_1) + (b_2 + b_1)\sqrt{2}$$

Standard addition

neutral:

$$(a + b\cdot\sqrt{2}) + (1_a + 1_b\sqrt{2}) = a + b\sqrt{2} \quad 1-a$$

$$1-b\sqrt{2}$$

$$1_a + 1_b\sqrt{2} = 0 + 0\cdot\sqrt{2}$$

$$1 = 0 + 0\cdot\sqrt{2} \quad ; \quad 0 \in \mathbb{Q}$$

invers:

$$(\underline{a} + \underline{b}\sqrt{2}) + (\underline{\bar{a}} + \underline{\bar{b}}\sqrt{2}) = \underline{0} + \underline{0}\cdot\sqrt{2}$$

$$\bar{x} = -a + (-b)\sqrt{2} \left\{ \begin{array}{l} a + \bar{a} = 0 \qquad \bar{a} = -a \in \mathbb{Q} \\ b\cdot\sqrt{2} + \bar{b}\sqrt{2} = \sqrt{2}\cdot(b + \bar{b}) = 0\cdot\sqrt{2} \\ \bar{b} = -b \in \mathbb{Q} \end{array} \right.$$

$\Rightarrow$ abelsche Gruppe