$x = a + b\sqrt{2} \quad ; \quad a, b \in \mathbb{Q}$

$*$: binär : $\quad x_1 \cdot x_2 \rightarrow x_3$

$$(a_1 + b_1\sqrt{2}) \cdot (a_2 + b_2\sqrt{2}) \rightarrow a_3 + b_3\sqrt{2} \quad ; \quad a_i, b_i \in \mathbb{Q}$$

$$a_1 a_2 + a_1 b_2 \sqrt{2} + a_2 b_1 \sqrt{2} + 2 \cdot b_1 b_2$$

$$\underbrace{(a_1 a_2 + 2 b_1 b_2)}_{a_3} + \underbrace{(a_1 b_2 + a_2 b_1)}_{b_3}\sqrt{2}$$

$$(x_1 \cdot x_2) \cdot x_3$$

asso : $\quad [(a_1 + b_1\sqrt{2}) \cdot (a_2 + b_2\sqrt{2})] \cdot (a_3 + b_3\sqrt{2}) =$

$$(a_1 + b_1\sqrt{2}) \cdot [(a_2 + b_2\sqrt{2}) \cdot (a_3 + b_3\sqrt{2})]$$

$$x_1 \cdot (x_2 \cdot x_3)$$

Komm : $\quad \underbrace{(a_1 + b_1\sqrt{2})}_{x_1} \cdot \underbrace{(a_2 + b_2\sqrt{2})}_{x_2} = \underbrace{(a_2 + b_2\sqrt{2})}_{x_2} \cdot \underbrace{(a_1 + b_1\sqrt{2})}_{x_1}$

Standardmultiplikation

neutral :

$$(a+b\sqrt{2}) \cdot \overbrace{(x+y\sqrt{2})}^{1} = a+b\sqrt{2}$$

$$\underbrace{(ax+2by)}_{a} + \underbrace{(xb+ya)}_{b} \cdot \sqrt{2}$$

$$\left| \begin{array}{l} ax + 2by = a \\ xb + ya = b \end{array} \right| \Rightarrow x = \frac{a-2by}{a} \qquad x = 1$$

$$\frac{a-2by}{a} \cdot b + ya = b \qquad | \cdot a$$

$$ab - 2b^2y + ya^2 = ab \qquad | -ab$$

$$y(a^2 - 2b^2) = 0 \qquad | : (\,)$$

$$y = 0$$

$$1 = 1 + 0 \cdot \sqrt{2}$$

invers: $\quad x \cdot \bar{x} = 1$

$$(a + b\sqrt{2}) \cdot (x + y\sqrt{2}) = 1 + 0 \cdot \sqrt{2}$$

$$\underbrace{(ax + 2by)}_{1} + \underbrace{(bx + ay)}_{0} \cdot \sqrt{2} = 1 + 0 \cdot \sqrt{2}$$

$$\left| \begin{array}{l} ax + 2by = 1 \\ bx + ay = 0 \end{array} \right| \longrightarrow \quad x = \frac{-ay}{b}$$

$$\frac{-a^2 y}{b} + \frac{2b^2 y}{b} = 1$$

$$y \cdot \frac{(2b^2 - a^2)}{b} = 1$$

$$y = b/_{2b^2 - a^2}$$

$$x = -\frac{a}{b} \cdot \frac{b}{2b^2 - a^2}$$

$$x = -\frac{a}{2b^2 - a^2}$$