$(\mathbb{N}, \cdot) \to a \cdot b = b \cdot a$ Kommutativ

abelsche

Halbgruppe

$\to (a \cdot b) \cdot c = a \cdot (b \cdot c)$ assoziativ

$abc = abc$; $a, b, c \in \mathbb{N}$

$\to \mathbb{N} \cdot \mathbb{N} \to \mathbb{N}$ binäre Operation

Monoid

$\to a \cdot \underline{1} = a \quad | : a <> 0$

neutrale Element

$\underline{1} = 1 \in \mathbb{N}$

$\to a \cdot \bar{a} = \underline{1} = 1$ inverses Element

$\bar{a} = 1/a \notin \mathbb{N}$

$(\mathbb{Q}, \sim)$: $a \sim b = \frac{1}{2}a \cdot 3b$

$2 \sim 3 = \frac{1}{2} \cdot 2 \cdot 3 \cdot 3 = 1 \cdot 9 = 9$

Binär: $\mathbb{Q} \sim \mathbb{Q} = \frac{1}{2} \cdot \mathbb{Q} \cdot 3 \cdot \mathbb{Q} = \mathbb{Q} \cdot \mathbb{Q} \to \mathbb{Q}$ ✓

Kommutativ: $a \sim b = b \sim a$

$\frac{1}{2}a \cdot 3b = \frac{1}{2}b \cdot 3a$

$\frac{3}{2}ab = \frac{3}{2}ab$ ✓

abelsch

assoziativ: $(a \sim b) \sim c = a \sim (b \sim c)$

$$(\underbrace{\tfrac{1}{2}a \cdot 3b}_{d}) \sim c = a \sim (\underbrace{\tfrac{1}{2}b \cdot 3c}_{d})$$

$$\tfrac{1}{2} \cdot (\tfrac{1}{2}a \cdot 3b) \cdot 3c = \tfrac{1}{2}a \cdot 3(\tfrac{1}{2}b \cdot 3c)$$

$$\tfrac{9}{4}\, abc = \tfrac{9}{4}\, abc \quad \checkmark$$

=> abelsche Halbgruppe

neutral: $a \sim 1 = a$

$$\tfrac{1}{2}a \cdot 3 \cdot 1 = a \qquad \Big| \cdot \tfrac{2}{3} \cdot \tfrac{1}{a} \;;\quad a <> 0$$

$$1 = \tfrac{2}{3} \in \mathbb{Q} \qquad \tfrac{1}{2} \cdot 0 \cdot 3 \cdot \tfrac{2}{3} = 0 \quad \checkmark$$

$$0 \sim 1 = 0$$

=> abelsche Monoid

inves: $a \sim \overline{a} = 1$

$\frac{1}{2}a \cdot 3\overline{a} = \frac{2}{3} \quad | \cdot \frac{2}{3} \cdot \frac{1}{a} \; ; \; a <> 0$

$\overline{a} = \frac{4}{9a} \in \mathbb{Q}$

$a = 0 : \quad \overline{a} = \frac{4}{9 \cdot 0} \quad \lightning$

=> Es existiert kein inverses Element

=> $(\mathbb{Q} \setminus \{0\}, \sim)$

=> abelsche Gruppe

$M = \{ e^x ; x \in \mathbb{Z} \}$ $\quad \#: e^x \# e^y = e^{2x+y}$

$e^{\text{ganze Zahl}}$ $\quad e^{\mathbb{Z}} \# e^{\mathbb{Z}} = e^{2\cdot\mathbb{Z}+\mathbb{Z}} = e^{\mathbb{Z}}$

Sinde :

assoziativ: $(e^x \# e^y) \# e^z = e^x \# (e^y \# e^z)$

$$e^{2x+y} \# e^z = e^x \# e^{2y+z}$$

$$e^{2\cdot(2x+y)+z} = e^{2x+(2y+z)}$$

$$e^{4x+2y+z} = e^{2x+2y+z} \quad ↯$$

$$e^x \# e^1 = e^x$$

$$e^{2x+1} = e^x \Rightarrow 1 = e^{-x}$$