2) $A = \{1; -1\}$ $\quad (A, *)$

$$\left.\begin{array}{ll} 1 \cdot 1 & = 1 \\ 1 \cdot (-1) & = -1 \\ (-1) \cdot 1 & = -1 \\ (-1) \cdot (-1) & = 1 \end{array}\right\} \in A \quad \Rightarrow \text{binär}$$

asso + komm., da Standardmultiplikation

neutral $\quad \left.\begin{array}{l} 1 \cdot 1 = 1 \\ (-1) \cdot 1 = -1 \end{array}\right\} \mathbb{1} = 1 \in A$

invers: $\quad \left.\begin{array}{ll} 1 \cdot \bar{x} = 1 & \Rightarrow \bar{x} = 1 \in A \\ (-1) \cdot \bar{x} = 1 & \Rightarrow \bar{x} = -1 \in A \end{array}\right\} \bar{x} = x$

2) $M = \{ a + b \cdot \sqrt{2} ; \; a, b \in \mathbb{Q} \}$ $\qquad (M, +, *)$

binär:

$$\left.\begin{aligned} x_1 &= a_1 + b_1\sqrt{2} \\ x_2 &= a_2 + b_2 \cdot \sqrt{2} \end{aligned}\right\} +$$

$$\left.\begin{aligned} &(a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2}) \\ &(a_1 + a_2) + (b_1 + b_2) \cdot \sqrt{2} \\ &\underbrace{\qquad}_{a_3} + \underbrace{\qquad}_{b_3} \cdot \sqrt{2} \end{aligned}\right\} a_i \, ; \, b_i \in \mathbb{Q}$$

assoz:

$$(x_1 + x_2) + x_3 = x_1 + (x_2 + x_3)$$

$$\left[(a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2})\right] + x_3 = x_1 + \left[(a_2 + b_2\sqrt{2}) + (a_3 + b_3\sqrt{2})\right]$$

$$\left[(a_1 + a_2) + (b_1 + b_2)\sqrt{2}\right] + (a_3 + b_3\sqrt{2}) = (a_1 + b_1\sqrt{2}) + \left[(a_2 + a_3) + (b_2 + b_3)\sqrt{2}\right]$$

$$(a_1 + a_2 + a_3) + (b_1 + b_2 + b_3) \cdot \sqrt{2}$$

neutral: $x + 1 = x$

$$(a + b\cdot\sqrt{2}) + (a_1 + b_1\sqrt{2}) = a + b\cdot\sqrt{2}$$
$$\underline{(a + a_1)} + \underline{(b + b_1)}\cdot\sqrt{2} = \underline{a} + \underline{b}\sqrt{2}$$

$$\left.\begin{array}{ll} a + a_1 = a & a_1 = 0 \\ b + b_1 = b & b_1 = 0 \end{array}\right\} \quad 1 = 0 + 0\cdot\sqrt{2} \in M$$

invers: $x + \bar{x} = 1$

$$(a + b\sqrt{2}) + (\bar{a} + \bar{b}\sqrt{2}) = 0 + 0\sqrt{2}$$

$$a + \bar{a} = 0 \quad \wedge \quad b + \bar{b} = 0$$
$$\bar{a} = -a \quad \wedge \quad \bar{b} = -b$$

$$\bar{x} = -a - b\sqrt{2} \in M$$

=> abelsche Gruppe