$(\mathbb{Z}, +)$

↳ Standardaddition

Sihdu: $M \times M \to M$

$a + b = c \quad ; a, b, c \in \mathbb{Z}$

$\mathbb{Z} + \mathbb{Z} \to \mathbb{Z}$

$M = \{ x \in \mathbb{Z} \mid x = 2k+1 ; k \in \mathbb{Z} \}$

Kommutativ: $a + b = b + a \quad ; a, b \in \mathbb{Z}$

assoziativ: $(a+b)+c = a + (b+c), \quad a, b, c \in \mathbb{Z}$

neutrales Element: $a + 1 = a$

Einselement

$1 = 0 \in \mathbb{Z}$

inverse Element: $a + \bar{a} = 1$

$a + \bar{a} = 0$

$\bar{a} = -a \in \mathbb{Z}$

$(10^x ; \cdot) ; x \in \mathbb{Q}$

$10^{1/2} ; 10^2 ; 10^{-1}$

binär.

$$10^x \cdot 10^y = 10^z$$
$$10^{x+y} = 10^z$$
$$x+y = z$$

$\Big\}\; x, y, z \in \mathbb{Q}$

assoziativ:

$$\underline{(10^x \cdot 10^y) \cdot 10^z} = 10^{x+y} \cdot 10^z$$
$(a \circ b) \circ c = a \circ (b \circ c)$
$$= 10^{(x+y)+z}$$
$$= 10^{x+(y+z)}$$
$$= 10^x \cdot 10^{y+z}$$
$$= \underline{10^x \cdot (10^y \cdot 10^z)}$$

kommutativ: $10^x \cdot 10^y = 10^{x+y}$

$$= 10^{y+x} = 10^y \cdot 10^x$$

neutrales Element:

$$10^x \cdot \underbrace{10^y}_{1} = 10^x$$

$$10^{x+y} = 10^{x+0} = 10^x$$

da $0 \in \mathbb{Q}$, gilt $1 = 10^0$

inverse Element:

$$10^x \cdot 10^z = 1$$

$$10^{x+z} = 10^0 \qquad z = -x \in \mathbb{Q}$$

$$\overline{x} = 10^{-x}$$

$\Rightarrow$ abelsche Gruppe

$(M, \heartsuit)$ ist abelsche Gruppe ?

Warum ist $a \heartsuit x = b$ immer lösbar? $| \heartsuit \bar{a}$

inverse

$(a \heartsuit x) \heartsuit \bar{a} = b \heartsuit \bar{a}$

$a \heartsuit (x \heartsuit \bar{a})$

abelsch

$\bar{a} \heartsuit (a \heartsuit x) = b \heartsuit \bar{a}$

assoziativ

$\underbrace{(\bar{a} \heartsuit a)} \heartsuit x = b \heartsuit \bar{a}$

$1 \heartsuit x = b \heartsuit \bar{a}$

neutral

$x = b \heartsuit \bar{a}$

binär

$\mathbb{L} = \{ b \heartsuit \bar{a} \}$