

(Menge, Operator) \rightarrow Sinne Operation
 $a + b \in \mathbb{Z}$

(\mathbb{Z} , +) \rightarrow Kommutativgesetz?

$$a + b = b + a ; a, b \in \mathbb{Z}$$

$$M = \{x \in \mathbb{Z} \mid x = 2k+1; k \in \mathbb{Z}\}$$

(M , +)

\rightarrow Assoziativgesetz?

$$(a+b)+c = a+(b+c) ; a, b, c \in \mathbb{Z}$$

\rightarrow neutrale Element

$$a + 1 = a \quad 1 = 0 \in \mathbb{Z}$$

\rightarrow inverse Element

$$a + \bar{a} = 1 \quad \bar{a} = -a \in \mathbb{Z}$$

$$(10^x, \cdot) ; x \in \mathbb{Q}$$

$$10^x \cdot 10^y = 10^{x+y}$$

Sinn: $10^x \cdot 10^y = 10^{x+y} = 10^z$

$$\left. \begin{array}{l} x+y = z \\ \mathbb{Q} + \mathbb{Q} \rightarrow \mathbb{Q} \end{array} \right\} x, y, z \in \mathbb{Q}$$

assoziativ: $(10^x \cdot 10^y) \cdot 10^z = 10^{x+y} \cdot 10^z$

Standard-Addition \rightarrow

$$\begin{aligned} &= 10^{(x+y)+z} = 10^{x+(y+z)} \\ &= 10^x \cdot 10^{y+z} \\ &= 10^x \cdot (10^y \cdot 10^z) \end{aligned}$$

Kommutativ: $10^x \cdot 10^y = 10^{x+y} = 10^{y+x} = 10^y \cdot 10^x$

neutrales Element: $10^x \cdot 10^{x_1} = 10^x$

$$10^{x+x_1} = 10^x$$

$$x + x_1 = x$$

$$x_1 = 0 \in \mathbb{Q}$$

$$\Rightarrow 1 = 10^0$$

inverses Element: $10^x \cdot 10^{x_2} = 1 = 10^0$

$$x + x_2 = 0 \Leftrightarrow x_2 = -x \in \mathbb{Q}$$

$$\bar{x} = 10^{-x}$$

\Rightarrow abelsche Gruppe

$(M, \#)$ ist abelsche Gruppe?

Ist die Gleichung $a \# x = b$ lösbar?

$$a \# x = b$$

$$| \# \bar{a}$$

inverse

$$(a \# x) \# \bar{a} = b \# \bar{a}$$

$$\bar{a} \# (a \# x) = b \# \bar{a}$$

$$(\bar{a} \# a) \# x = b \# \bar{a}$$

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$$1 \# x = b \# \bar{a}$$

$$x = b \# \bar{a}$$

Kommutativ

assoziativ

neutral