S 145 Nr. 1)

NS $f(x) = x^3 - 3x^2 - 4x = x(x-4)(x+1)$ $\nearrow$ $x_1 = -1$, $x_2 = 0$, $x_3 = 4$

Integral $\int_{-1}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$

Stammf. $F(x) = \frac{1}{4}x^4 - x^3 - 2x^2$

$F(-1) = \frac{1}{4} + 1 - 2 = -\frac{3}{4}$

Grenzen $F(0) = 0 + 0 + 0 = 0$

$F(4) = 4^3 - 4^3 - 2 \cdot 16 = -32$

Differenz

$$\left.\begin{array}{l} F(0) - F(-1) = 0 - \left(-\frac{3}{4}\right) = \frac{3}{4} \\ |F(4) - F(0)| = |-32 - 0| = |-32| = 32 \end{array}\right\} + \underline{32\frac{3}{4}}$$

2) a) $f(x) = 7x \boxed{-\ 2} \cdot e^{3x-4} \quad \longrightarrow \quad G(x) = e^{3x-4}$

$g(x) = \underline{3} \cdot e^{3x-4}$

$H(x) = \frac{7}{2}x^2 - \frac{2}{3}\, e^{3x-4}$

b) $f(x) = \boxed{4} \cdot (5-3x)^3 \quad \longrightarrow \quad G(x) = (5\underline{-3}x)^4 \checkmark$

$g(x) = \underline{-12}\,(5-3x)^3$

$K(x) = -\frac{1}{3}\,(5-3x)^4$

3) b) $f(x) = \sqrt{5-6x} \quad \wedge \quad g(x) = x$

$\sqrt{5-6x} = x \quad |^2$

$5 - 6x = x^2 \quad \Leftrightarrow \quad x^2 + 6x - 5 = (x-3)(x-?)$

$\int_2^3 f(x) - g(x)\, dx$

3) 5)

$$\sqrt{5x-6} = x \quad \uparrow^2$$

$$5x-6 = x^2 \iff x^2-5x+6 = (x-2)(x-3) = 0$$

$$\int_2^3 f(x) - g(x)\, dx = \int_2^3 (\sqrt{5x-6} - x)\, dx$$

$$F(x) = \tfrac{2}{15}(5x-6)^{3/2} - \tfrac{1}{2}x^2$$

$$F(3) = \tfrac{2}{15} \cdot 9^{3/2} - \tfrac{1}{2} \cdot 9 = \frac{54}{15} - \tfrac{9}{2}$$

$$F(2) = \tfrac{2}{15} \cdot 4^{3/2} - \tfrac{1}{2} \cdot 4 = \frac{16}{15} - 2$$

$$F(3) - F(2) = \frac{54}{15} - \frac{9}{2} - \left(\frac{16}{15} - 2\right)$$

$$= \frac{54}{15} - \frac{16}{15} - \frac{9}{2} + \frac{4}{2} = \frac{38}{15} - \frac{5}{2}$$

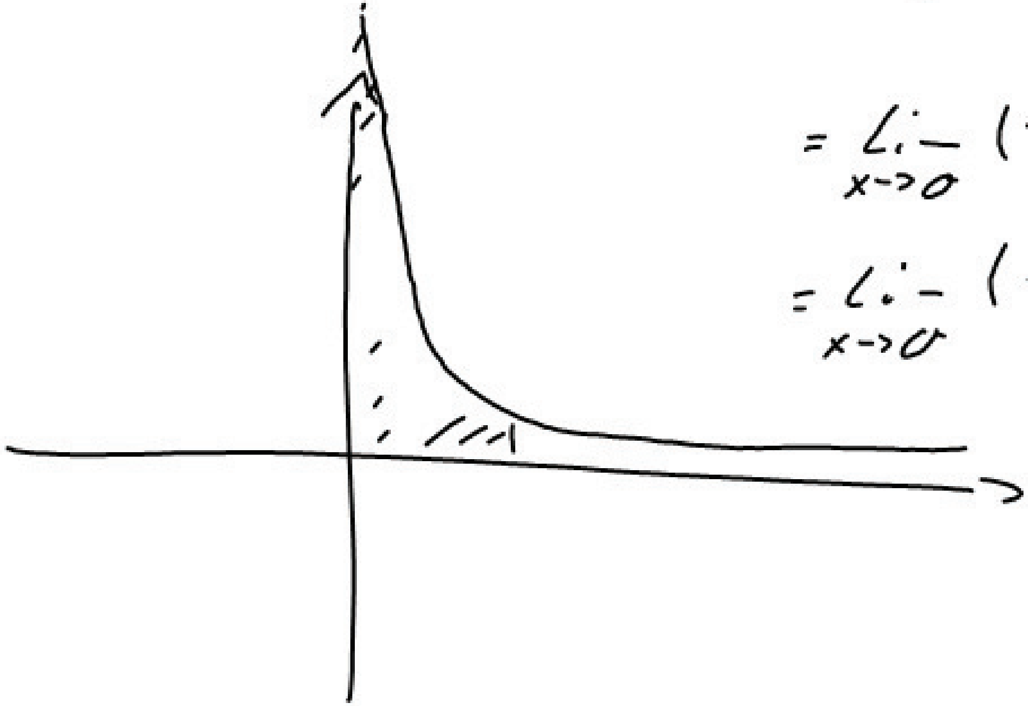
$$= \frac{76 - 75}{30} = \underline{\underline{\tfrac{1}{30}}}$$

3) a)

$f(x) = 1/x^2 \; ; \; g(x) = 1/x \quad \Rightarrow \quad 1/x^2 = 1/x \quad \uparrow^{-1}$

$x^2 = x \quad \Leftrightarrow \quad x^2 - x = x(x-1) = 0 \quad \rightarrow \quad \begin{matrix} x_1 = 0 \\ x_2 = 1 \end{matrix}$

$$\int_0^1 \left(1/x^2 - 1/x \right) dx = \left[-\frac{1}{x} - \ln x \right]_0^1$$



$$= \lim_{x \to 0} \left(-\frac{1}{1} - \ln 1 \right) - \left(-\frac{1}{x} - \ln x \right)$$

$$= \lim_{x \to 0} \left(-1 + 1/x + \ln x \right) \rightarrow \infty$$

S. 149 Nr. 2

$$\int_{\alpha}^{\boxed{\infty}} (2x-2)^{-2}\,dx = 1/16$$

$$F(x) = -\tfrac{1}{2}(2x-2)^{-1} = \frac{-1}{4x-4}$$

$$\left.\begin{aligned} F(\infty) &= \lim_{x\to\infty} \\ F(\alpha) &= -\frac{1}{4\alpha-4} \end{aligned}\right\} \lim_{x\to\infty} \underbrace{-\frac{1}{4x-4}}_{1/\infty = 0} - \left(-\frac{1}{4\alpha-4}\right) = 1/16$$

$$\frac{1}{4\alpha-4} = 1/16 \qquad ()^{-1}$$

$$4\alpha - 4 = 16$$

$$4\alpha = 20$$

$$\alpha = 5$$

Beispiel: $\int x^2 \cdot \sin(3x)\, dx$

$g(x)$ $\quad$ $f'(x)$

$f'(x) = \sin(3x) \quad \rightarrow \quad f(x) = -\frac{1}{3}\cos(3x)$

$g(x) = x^2 \quad \rightarrow \quad g'(x) = 2x$

$-\frac{1}{3} x^2 \cdot \cos(3x) - \int -\frac{1}{3}\cos(3x) \cdot 2x \, dx$

$-\frac{1}{3} x^2 \cdot \cos(3x) + \frac{2}{3} \int x \cdot \cos(3x)\, dx$

$f'(x) = \cos(3x) \quad \rightarrow \quad f(x) = \frac{1}{3}\sin(3x)$

$g(x) = x \quad \rightarrow \quad g'(x) = 1$

$\frac{1}{3} x \cdot \sin(3x) - \int \frac{1}{3}\sin(3x)\, dx$

$\frac{1}{3} x \cdot \sin(3x) - \frac{1}{3} \cdot \int \sin(3x)\, dx$

$\frac{1}{3} x \cdot \sin(3x) - \frac{1}{3} \cdot \left(-\frac{1}{3}\cos(3x)\right)$

$-\frac{1}{3} x^2 \cdot \cos(3x) + \frac{2}{9} x \sin(3x) + \frac{2}{27}\cos(3x)$

$$\int \tfrac{1}{3} \cdot x^2 \cdot e^{4-3x} \, dx = \tfrac{1}{3} \int x^2 \cdot e^{4-3x} \, dx$$

$$f'(x) = e^{4-3x} \quad \rightarrow \quad f(x) = -\tfrac{1}{3} e^{4-3x}$$

$$g(x) = x^2 \quad \rightarrow \quad g'(x) = 2x$$

$$-\tfrac{1}{3}x^2 e^{4-3x} + \tfrac{2}{3} \int x \cdot e^{4-3x} \, dx$$

$$f'(x) = e^{4-3x} \quad \rightarrow \quad f(x) = -\tfrac{1}{3} e^{4-3x}$$

$$g(x) = x \quad \rightarrow \quad g'(x) = 1$$

$$-\tfrac{1}{3}x \cdot e^{4-3x} - \tfrac{1}{3} \int e^{4-3x} = \boxed{-\tfrac{1}{3}x e^{4-3x} + \tfrac{1}{9} e^{4-3x}}$$

$$\tfrac{1}{3}\left[-\tfrac{1}{3}x^2 e^{4-3x} + \tfrac{2}{3} \cdot \left(-\tfrac{1}{3}x e^{4-3x} + \tfrac{1}{9} e^{4-3x}\right)\right]$$

$$-\tfrac{1}{9} e^{4-3x} \left(x^2 + \tfrac{2}{3}x - \tfrac{2}{9}\right) + C$$

$$\int \cos(2x) \cdot e^{4-2x}\, dx$$

I

$$f'(x) = e^{4-2x} \quad \rightarrow \quad f(x) = \boxed{-1/2}\, e^{4-2x}$$

$$g(x) = \cos(2x) \quad \rightarrow \quad g'(x) = \boxed{-2}\, \sin(2x)$$

$$-1/2\, e^{4-2x} \cdot \cos(2x) - \cancel{1} \int e^{4-2x} \cdot \sin(2x)\, dx$$

II

$$f'(x) = e^{4-2x} \quad \rightarrow \quad f(x) = \boxed{-1/2}\, e^{4-2x}$$

$$g(x) = \sin(2x) \quad \rightarrow \quad g'(x) = \boxed{2}\, \cos(2x)$$

(-1)

$$-1/2\, e^{4-2x} \cdot \sin(2x) + \int e^{4-2x} \cdot \cos(2x)\, dx$$

$$-1/2\, e^{4-2x} \cdot \cos(2x) - \left[-1/2\, e^{4-2x} \cdot \sin(2x) + \int e^{4-2x} \cdot \cos(2x)\, dx \right]$$

$$1/2\, e^{4-2x} \left(\sin(2x) - \cos(2x) \right) - \int e^{4-2x} \cdot \cos(2x)\, dx$$

$$\int \cos(2x)\cdot e^{4-2x} = \frac{1}{2} e^{4-2x} \cdot (\sin(2x) - \cos(2x)) - \int e^{4-2x} \cdot \cos(2x)\,dx$$

$$2 \cdot \int \cos(2x)\, e^{4-2x} = \frac{1}{2} \cdot e^{4-2x} (\sin(2x) - \cos(2x))$$

$$\int \cos(2x)\, e^{4-2x}\,dx = \frac{1}{4} e^{4-2x} \cdot (\sin(2x) - \cos(2x)) + C$$