S. 94 Nr. 1

$$\lim_{x \to (-4)} \frac{2(x+4)}{\underbrace{\sqrt{8-2x}}_{a} - \underbrace{(8+x)}_{b}} \cdot \frac{\sqrt{8-2x}+(8+x)}{\underbrace{\sqrt{8-2x}}_{a} + \underbrace{(8+x)}_{b}}$$

$$\lim_{x \to (-4)} \frac{2(x+4)\left(\sqrt{8-2x}+(8+x)\right)}{\underbrace{(8-2x)}_{a^2} - \underbrace{(8+x)^2}_{b^2}} \longrightarrow 8-2x-(64+16x+x^2)$$

$$-x^2-18x-56$$

$$-(x^2+18x+56)$$

$$\lim_{x \to (-4)} \frac{2 \cdot \cancel{(x+4)} \cdot \left(\sqrt{8-2x}+(8+x)\right)}{-\cancel{(x+4)}\,(x+14)} = \left[\frac{2 \cdot (8)}{-10}\right] = -\frac{8}{5}$$

2)

$$\lim_{x \to 4} \frac{x^3 - 2x^2 - 12x + 16}{x^2 + x - 20} = \left[\frac{0}{0}\right]$$

L'Hospital : $\Rightarrow \displaystyle\lim_{x \to 4} \frac{3x^2 - 4x - 12}{2x + 1} = \frac{20}{9} = 2\,{}^2/_9$

$$\begin{aligned}
&(x^3 - 2x^2 - 12x + 16) : (x - 4) = \underline{x^2 + 2x - 4} \\
&\underline{-(x^3 - 4x^2)} \\
&\qquad 2x^2 - 12x + 16 \\
&\qquad \underline{-(2x^2 - 8x)} \\
&\qquad\qquad -4x + 16 \\
&\qquad\qquad \underline{-(-4x + 16)} \\
&\qquad\qquad\quad —
\end{aligned}$$

$$\lim_{x \to 4} \frac{\cancel{(x-4)} \cdot (x^2 + 2x - 4)}{(x - 4)(x + 5)} = \frac{20}{9} = 2\,{}^2/_9$$

3)

$$\lim_{x \to \infty} \underbrace{\left(1+\frac{5}{x}\right)^x} - \frac{3}{\sqrt[x]{9}} + \left(\frac{2}{4} \cdot \frac{\sin x}{x}\right)^2$$

$$e^5 - \frac{3}{1} + \left(\frac{1}{2} \cdot \underbrace{\frac{[-1;1]}{\infty}}\right)^2$$

$$\underbrace{\left(1/2 \cdot 0\right)^2}_{0}$$

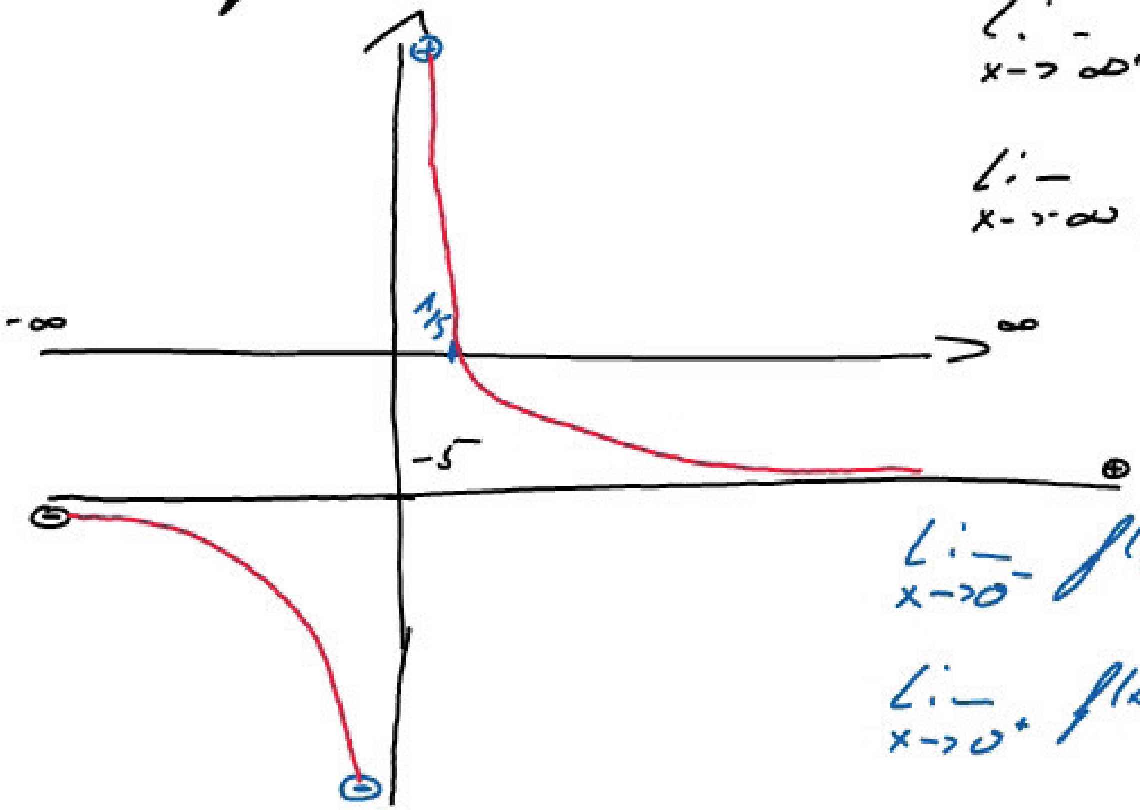
$$\to e^5 - 3$$

$$e^{i\pi} = (-1)$$

Asymptoten

-> graphische Interpretation eines Grenzwertes

$$f(x) = \frac{1}{x} - 5$$



$$\lim_{x \to \infty} f(x) = \frac{1}{\infty} - 5 = -5^{+}$$

$$\lim_{x \to -\infty} f(x) = \frac{1}{-\infty} - 5 = -5^{-}$$

$$f(x) = 0$$

$$\lim_{x \to 0^-} f(x) = [-\infty - 5] = -\infty$$

$$\lim_{x \to 0^+} f(x) = [\infty - 5] = \infty$$

$$f(x) = \frac{2}{3-x} + 4 \quad ; \quad D = \mathbb{R} \setminus \{3\}$$

$$\lim_{x \to \infty} f(x) = \left[\frac{2}{-\infty} + 4\right] = 4^-$$

$$\lim_{x \to 3^-} f(x) = \left[\frac{2}{0^+} + 4\right] = \infty$$

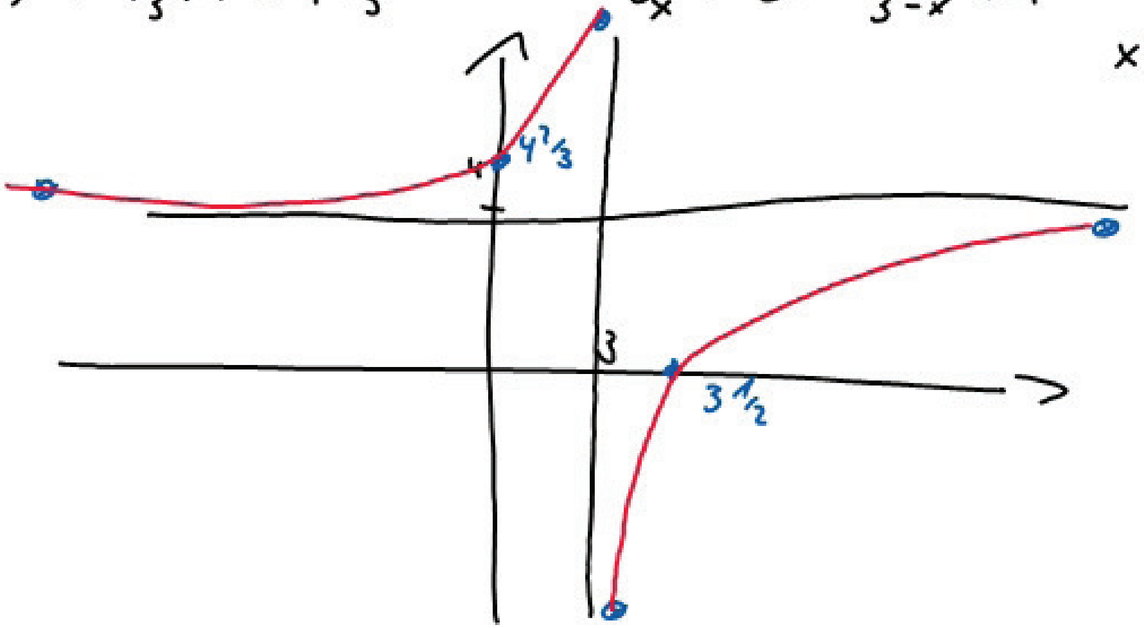
$$\lim_{x \to -\infty} f(x) = \left[\frac{2}{\infty} + 4\right] = 4^+$$

$$\lim_{x \to 3^+} f(x) = \left[\frac{2}{0^-} + 4\right] = -\infty$$

$S_y : f(0) = \frac{2}{3} + 4 = 4\frac{2}{3}$

$S_x : 0 = \frac{2}{3-x} + 4 \qquad 3 - x = -\frac{1}{2}$

$x = 3\frac{1}{2}$



S 100 Nr. 1

$$f(x) = \frac{(x-2)(x+2)(x-2)}{2\cdot(x-2)(x-2)(x+3)} \quad ; \mathbb{D} = \mathbb{R} \setminus \{-3;2\}$$

$$f_e(x) = \frac{x+2}{2\cdot(x+3)} \quad ; \mathbb{D} = \mathbb{R} \setminus \{-3\}$$

2

hebbare Lücke

1. $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f_e(2) = 2/5 \qquad P(2/\,^2/_5)$

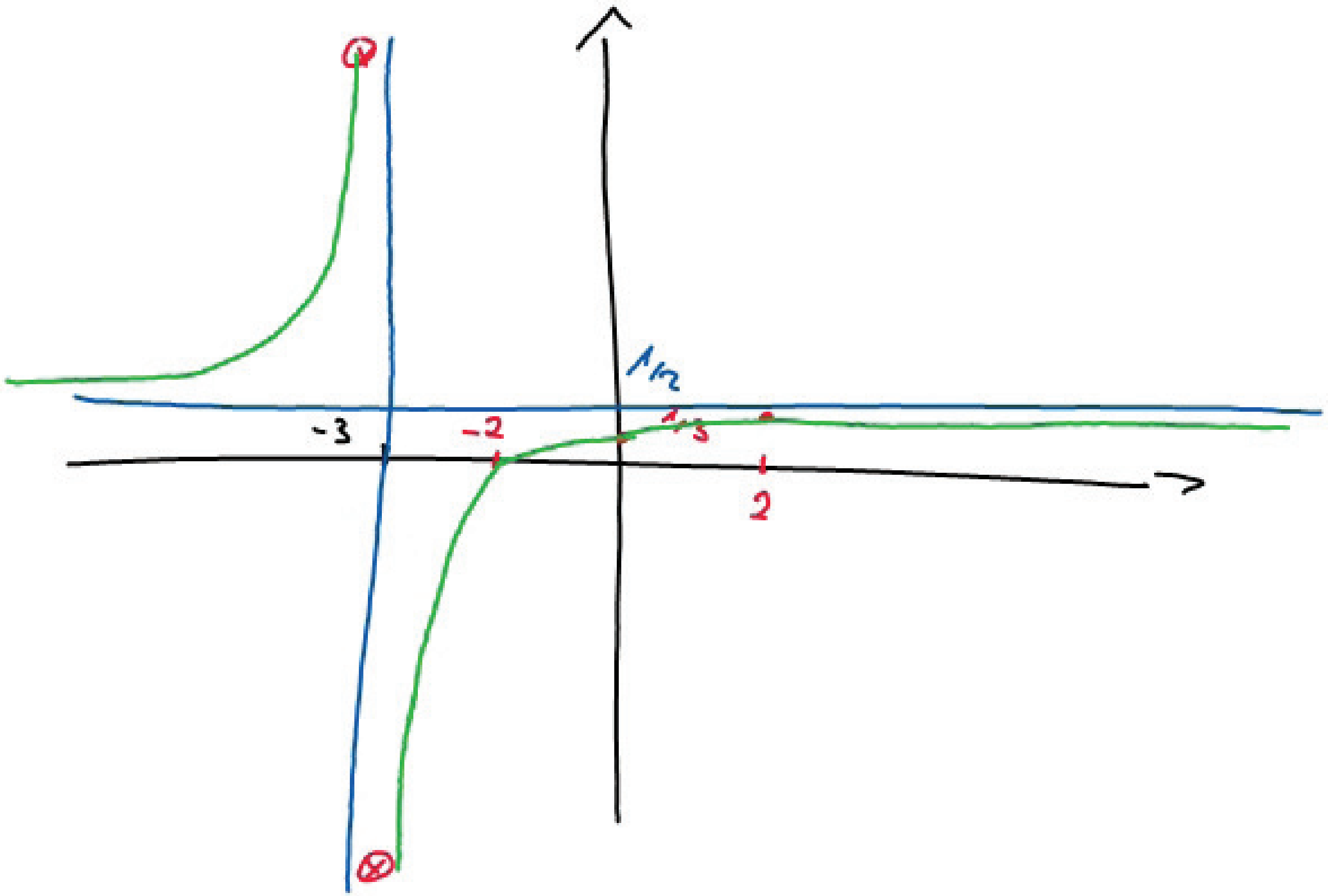
2. $\lim\limits_{x\to -3^-} f(x) = \left[\frac{-1}{-3^- + 3}\right] = \left[\frac{-1}{0^-}\right] = \infty$

$\lim\limits_{x\to -3^+} f(x) = \left[\frac{-1}{-3^+ + 3}\right] = \left[\frac{-1}{0^+}\right] = -\infty$

senkrechte Asymp.

3. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x(1+\overbrace{\tfrac{2}{x}}^{\to 0})}{x\cdot(2+\underbrace{6/x}_{\to 0})} = 1/2 = \lim\limits_{x\to -\infty} f(x)$

$S_x: f(x) = 0 \Rightarrow (-2/0) \qquad S_y: f(0) \Rightarrow (0/\tfrac{1}{3})$



1/2
-3
-2
1/3
2

2) $f(x) = \dfrac{(x-4)(x-5)(x+2)}{(x-4)\ (x+3)}$; $\mathbb{D} = \mathbb{R} \setminus \{-3; 4\}$

$f_e(x) = \dfrac{(x-5)(x+2)}{x+3}$; $\mathbb{D} = \mathbb{R} \setminus \{-3\}$

$\lim\limits_{x \to 4^+} f(x) = \lim\limits_{x \to 4^-} f(x) = f_e(4) = -6/7 \to P(4/-6/7)$

$\lim\limits_{x \to -3^-} f(x) = \left[\dfrac{8}{0^-}\right] = -\infty$

$\lim\limits_{x \to -3^+} f(x) = \left[\dfrac{8}{0^+}\right] = +\infty$

} senkrechte Asy.

$\lim\limits_{x \to \infty} f(x) = \dfrac{x^2 - 3x - 10}{x+3} = \dfrac{x^2(1 - \overbrace{\frac{3}{x} - \frac{10}{x^2}}^{0})}{x\ (1 + \underbrace{\frac{3}{x}}_{0})} = [x] = \infty$

diagonale Asymptote.

$$(x^2-3x-10):(x+3) = \boxed{x-6} + \frac{8}{x+3}$$

$$-(x^2+3x)$$

$$-6x-10$$

$$-(-6x-18)$$

$$8$$

$S_x: \; x_1 = 5 \; ; \; x_2 = -2$

$S_y: \; -10/3$