S. 133 Nr. 1 a) $\quad a \cdot \cos(bx + c) + d$

$$f(x) = \tfrac{2}{3}\left(6 - 4{,}5 \cdot \cos\left(\tfrac{1}{3}x - 4{,}5\pi\right)\right) - 4$$
$$= 4 - 3 \cdot \cos\left(\tfrac{1}{3}x - 4{,}5\pi\right) - 4$$
$$= -3 \cdot \cos\left(\tfrac{1}{3}x - 4{,}5\pi\right)$$

<u>Vereinfachung:</u>

$$f(x) = -3 \cdot \left[\cos\left(\tfrac{1}{3}x\right) \cdot \overbrace{\cos(4{,}5\pi)}^{0} + \sin\left(\tfrac{1}{3}x\right) \cdot \underbrace{\sin(4{,}5\pi)}_{1}\right]$$

$$f(x) = -3 \cdot \sin\left(\tfrac{1}{3}x\right) + 0$$

<u>Wertebereich:</u> Exponenten ≙ [illegible]

$$-3 \cdot [-1; 1] + 0 \Rightarrow y \in [-3; 3]$$

$$\left[\, -3 \cdot [-1; 1] + 4 = [-3; 3] + 4 = [1; 7] \,\right]$$

Symmetrie $f(x) = -f(-x)$

$$-3\cdot\sin\left(\tfrac{1}{3}x\right) = -\left[-3\cdot\sin\left(-\tfrac{1}{3}x\right)\right] \quad \Big|\cdot\left(-\tfrac{1}{3}\right)$$

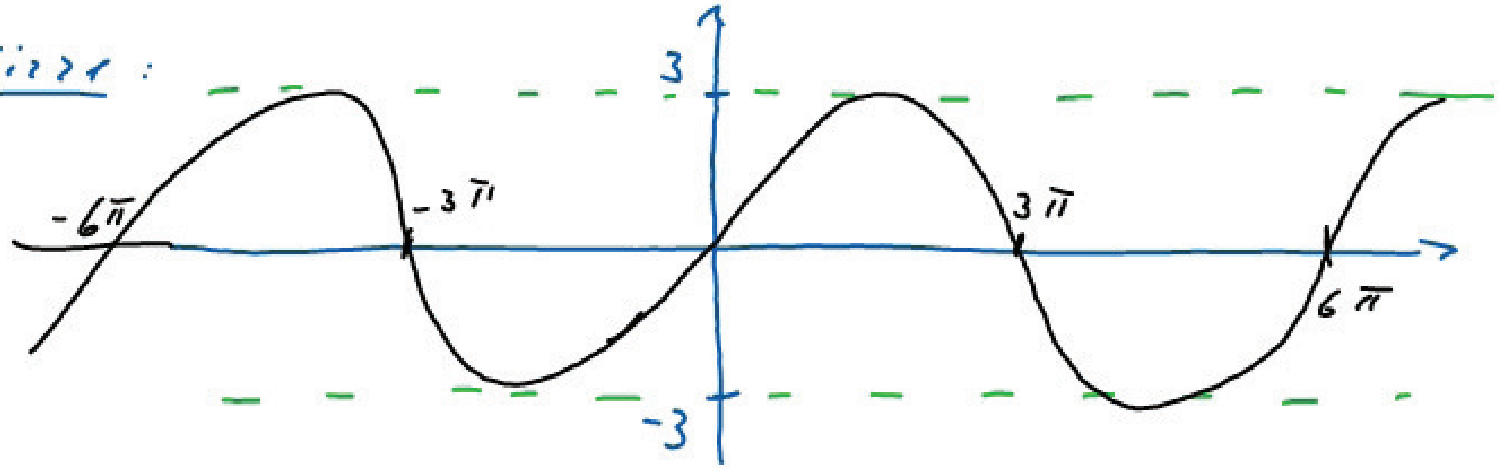
$$\sin\left(\tfrac{1}{3}x\right) = -\sin\left(-\tfrac{1}{3}x\right) \quad \Big\}\ \alpha = \tfrac{1}{3}x$$

$$\sin(\alpha) = -\sin(-\alpha) \quad \checkmark$$

Periode: $P_{neu} = \frac{2\pi}{1/3} = 6\pi \Rightarrow f(x) = f(x+6\pi)$

$$\begin{aligned} f(x+6\pi) &= -3\cdot\sin\left(\tfrac{1}{3}\cdot(x+6\pi)\right) \\ &= -3\cdot\sin\left(\tfrac{1}{3}x+2\pi\right) \\ &= -3\cdot\left[\sin\left(\tfrac{1}{3}x\right)\cdot\underbrace{\cos(2\pi)}_{1} + \cos\left(\tfrac{1}{3}x\right)\cdot\underbrace{\sin(2\pi)}_{0}\right] \\ &= -3\cdot\sin\left(\tfrac{1}{3}x\right) = f(x) \quad \checkmark \end{aligned}$$

Skizze:



$$\sin^6(2x+\pi) = [\underbrace{\sin(2x+\pi)}_{\sin(2x)\cdot\cos(\pi)+\cos(2x)\cdot\sin(\pi)}]^6 = (-\sin(2x))^6 = \sin^6(2x)$$

b) $g(x) = -3\cdot[\cos(4x+\frac{3}{2}\pi)]^4 - 2$

$= -3\cdot[\cos(4x)\cdot\cos(\frac{3}{2}\pi) - \sin(4x)\cdot\sin(\frac{3}{2}\pi)]^4 - 2$

$= -3\cdot[\cos(4x)\cdot 0 - \sin(4x)\cdot(-1)]^4 - 2$

$= -3\cdot\sin^4(4x) - 2$

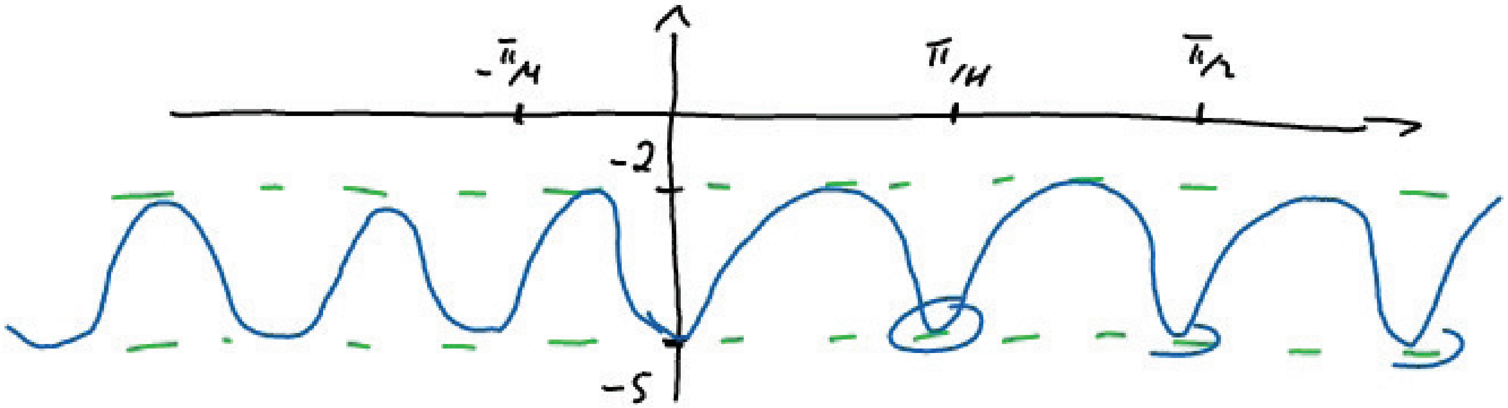
W: $-3\cdot[0;1] - 2 = [-3;0] - 2 = y\in[-5;-2]$

Symmetrie: $f(x) = f(-x)$

$$-3 \cdot \sin^4(4x) - 2 = -3 \cdot \sin^4(-4x) - 2$$
$$= -3 \cdot [\sin(-4x)]^4 - 2 \quad | \cdot (-\tfrac{1}{3}) \, | +2$$
$$\sin^4(4x) = [\sin(-4x)]^4 = [-\sin(4x)]^4$$
$$= \sin^4(4x)$$

Periode $P_{NEU} = \frac{\pi}{4}$ $\quad f(x) = f(x + \pi/4)$

$$f(x + \pi/4) = -3 \cdot \sin^4(4 \cdot (x + \pi/4)) - 2$$
$$= -3 \cdot [\sin(4x + \pi)]^4 - 2$$
$$= -3\,[\sin(4x) \cdot \underbrace{\cos(\pi)}_{(-1)} + \overbrace{\sin(\pi)}^{0} \cdot \cos(4x)]^4 - 2$$
$$= -3 \cdot [-\sin(4x)]^4 - 2$$
$$= -3 \cdot \sin^4(4x) - 2 = f(x) \checkmark$$



2) a) $h(x) = \ln(x^2-9) + 3/x$

$$h'(x) = \frac{1}{x^2-9} \cdot 2x + \left(-3/x^2\right)$$

$$[\ln \heartsuit]' = \frac{1}{\heartsuit} \cdot \heartsuit'$$

b) $k(x) = 42 \cdot (x^2-8x+12)^{-1/4}$

$$[(\heartsuit)^n]' = n \cdot \heartsuit^{n-1} \cdot \heartsuit'$$

$$k'(x) = -\frac{21}{2} \cdot (x^2-8x+12)^{-5/4} \cdot (2x-8)$$

$$= \frac{21(8-2x)}{2 \cdot \sqrt[4]{(x^2-8x+12)^5}}$$

c) $f(x) := 2 \cdot e^{\sqrt{3x-2} + \sin(\sqrt{2-x})}$

$[e^{\heartsuit}]' = e^{\heartsuit} \cdot \heartsuit'$

$$f'(x) = 2 \cdot e^{\sqrt{3x-2} + \sin(\sqrt{2-x})} \cdot \left[\underbrace{\left(\sqrt{3x-2} + \sin\sqrt{2-x}\right)'}_{\frac{3}{2\sqrt{3x-2}} + \cos\sqrt{2-x} \cdot \frac{-1}{2\cdot\sqrt{2-x}}} \right]$$

$$(a^x)' = \left[\left(e^{\ln a}\right)^x\right]' = \left[e^{\ln a \cdot x}\right]'$$

$$e^{\ln a \cdot x} \cdot \ln a = a^x \cdot \ln a$$

S 138 Nr. 1)

a) $F(x) = \frac{3}{2}x^2 - 2x^3 - x^5 + 12x + C$

b) $g(x) = 7x^{-2} - 3x^{-4} - \frac{1}{x}$

$G(x) = -7x^{-1} + x^{-3} - \ln x = -\frac{7}{x} + \frac{1}{x^3} - \ln x$

2) $g(x) = -(x^2 + 3x + 2) = -(x+2)(x+1)$

$\int_{-2}^{-1} (-x^2 - 3x - 2)\,dx = \left[-\frac{1}{3}x^3 - \frac{3}{2}x^2 - 2x \right]_{-2}^{-1} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

$F(-1) = \frac{1}{3} - \frac{3}{2} + 2 = \frac{2 - 9 + 12}{6} = \frac{5}{6}$

$F(-2) = \frac{8}{3} - 6 + 4 = \frac{2}{3} = \frac{4}{6}$