1) $$f(x) = \begin{cases} 4 - x^2 & ; x < 3 \\ x^2 + ax + b & ; x \geq 3 \end{cases} \quad ; \quad f'(x) = \begin{cases} -2x & ; x < 3 \\ 2x + a & ; x \geq 3 \end{cases}$$

2) $$g(x) = 3 \cdot [\cos(2x - \pi) + 2]$$

3) $$k(x) = 5 + 2/3 \cdot \cos^4\left(1/2 \cdot (\pi - x)\right)$$

stetig:

$$\left.\begin{aligned} &\lim_{x \to 3^-} f(x) = 4 - 3^2 = -5 \\ &\lim_{x \to 3^+} f(x) = f(x) = 3^2 + 3a + b \end{aligned}\right\} -5 = 9 + 3a + b$$

$$b = -14 - 3 \cdot (-12)$$
$$b = 22$$

diff:

$$\left.\begin{aligned} &\lim_{x \to 3^-} f'(x) = -6 \\ &\lim_{x \to 3^+} f'(x) = f'(3) = 6 + a \end{aligned}\right\} -6 = 6 + a$$

$$a = -12$$

$$g(x) = 3 \cdot \underbrace{\cos(2x - \pi)}_{} + 6$$

$$\cos(2x) \cdot \underbrace{\cos(\pi)}_{-1} + \sin(2x) \cdot \underbrace{\sin(\pi)}_{0}$$

$$g(x) = -3 \cdot \cos(2x) + 6$$

$\underline{W}$: $\quad -3 \cdot [-1;1] + 6 = [-3;3] + 6 \Rightarrow y \in [3;9]$

$$P_{NEU} = \frac{2\pi}{2} = \pi \quad \Rightarrow \quad f(x) = f(x+\pi)$$

$$-3 \cdot \cos(2x) + 6 = -3 \cdot \cos(2 \cdot (x+\pi)) + 6 \quad | -6 \quad \cdot \left(-\tfrac{1}{3}\right)$$

$$\cos(2x) = \cos(2x + 2\pi)$$

$$= \cos(2x) \cdot \underbrace{\cos(2\pi)}_{1} - \sin(2x) \cdot \underbrace{\sin(2\pi)}_{0}$$

$$= \cos(2x) \quad \checkmark$$

Symmetrie: $f(x) = f(-x)$

$$-3 \cdot \cos(2x) + 6 = -3 \cdot \cos(-2x) + 6 \quad | -6 \quad \cdot \left(-\tfrac{1}{3}\right)$$

$$\cos(2x) = \cos(-2x) \quad \checkmark$$

$$\cos(\alpha) = \cos(-\alpha)$$

$$K(x) = 5 + \tfrac{2}{3} \cdot \cos^4\left(\tfrac{1}{2} \cdot (x - \pi)\right) \qquad \cos^4(x) = [\cos(x)]^4$$

$$= 5 + \tfrac{2}{3} \cdot [\underbrace{\cos(\tfrac{1}{2}x - \tfrac{1}{2}\pi)}_{\cos(\frac{1}{2}x) \cdot \underbrace{\cos(\frac{\pi}{2})}_{0} + \sin(\frac{x}{2}) \cdot \underbrace{\sin(\frac{\pi}{2})}_{1}}]^4$$

$$\Rightarrow K(x) = \tfrac{2}{3} \cdot \sin^4\left(\tfrac{1}{2}x\right) + 5$$

$$W: \quad \tfrac{2}{3} \cdot [0; 1] + 5 = [0; \tfrac{2}{3}] + 5 \Rightarrow y \in [5; 5\tfrac{2}{3}]$$

$[-2;3]^2 = [4;9]$ f

$\downarrow$

$[-2;0]^2 \vee ]0;3]^2 = [4;0] \vee ]0;9]$

$[0;9]$

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Periode $\quad P_{neu} = \frac{\pi}{1/2} = 2\pi \Rightarrow f(x) = f(x+2\pi)$

$$\tfrac{2}{3} \cdot \sin^4(\tfrac{1}{2}x) + 5 = \tfrac{2}{3} \cdot \sin^4(\tfrac{1}{2}\cdot(x+2\pi)) + 5 \quad \Big| -5 \;\cdot \tfrac{3}{2}$$

$$\begin{aligned}
\sin^4(\tfrac{1}{2}x) &= [\sin(\tfrac{1}{2}x+\pi)]^4 \\
&= [\sin(\tfrac{1}{2}x)\cdot \underbrace{\cos(\pi)}_{-1} + \cos(\tfrac{1}{2}x)\cdot \underbrace{\sin(\pi)}_{0}]^4 \\
&= [-\sin(\tfrac{1}{2}x)]^4 \\
&= \sin^4(\tfrac{1}{2}x) \quad \checkmark
\end{aligned}$$

Symmetrie $\quad f(x) = f(-x) \quad$ Achsen

$$\tfrac{2}{3} \cdot \sin^4\left(\tfrac{1}{2}x\right) + 5 = \tfrac{2}{3} \cdot \sin^4\left(-\tfrac{1}{2}x\right) + 5 \quad \Big| -5 \cdot \tfrac{3}{2}$$

$$\sin^4\left(\tfrac{1}{2}x\right) = \left[\sin\left(-\tfrac{1}{2}x\right)\right]^4$$

$$= \left[-\sin\left(\tfrac{1}{2}x\right)\right]^4$$

$$= \sin^4\left(\tfrac{1}{2}x\right) \quad \checkmark$$