$$\left.\begin{array}{ll} 1) & \displaystyle\sum_{k=3}^{\infty} \frac{4}{2^{3k}} \\[2ex] 2) & \displaystyle\sum_{k=4}^{8} 9\cdot\left(\tfrac{2}{3}\right)^{k} \\[2ex] 3) & \displaystyle\sum_{k=3}^{\infty} \frac{2^{2k+1}}{3k!} \end{array}\right\} \underline{\text{WERT}}$$

$$4)\quad \sum_{k=1}^{\infty} (-3)^{k+1}\cdot\frac{2}{3^{4k}} \qquad !$$

$$5)\quad \sum_{k=1}^{\infty} \frac{\sqrt{k}\cdot(k+2)^2}{(3k)^k}$$

$$6)\quad \sum_{k=1}^{\infty} \frac{k^2\cdot 5^k}{(3k)!}$$

$$1)\quad 4\cdot\sum\frac{1}{2^{3k}} = 4\cdot\sum\frac{1}{(2^3)^k} = 4\cdot\sum\frac{1}{8^k} = 4\cdot\sum\left(\tfrac{1}{8}\right)^k$$

$$4\cdot\sum_{k=0}^{\infty}\left(\tfrac{1}{8}\right)^{k+3} = 4\cdot\sum_{k=0}^{\infty}\left(\tfrac{1}{8}\right)^{k}\cdot\left(\tfrac{1}{8}\right)^{3} = 4\cdot\left(\tfrac{1}{8}\right)^{3}\cdot\sum_{k=0}^{\infty}\left(\tfrac{1}{8}\right)^{k}$$

$$2^2\cdot\left(\frac{1}{2^3}\right)^3\cdot\frac{1}{1-1/8} = \frac{2^2}{2^9}\cdot\frac{8}{7} = \frac{1}{2^4}\cdot\frac{1}{7} = 1/112$$

2) $\sum_{k=4}^{8} 9 \cdot \left(\tfrac{2}{3}\right)^k = \sum_{k=0}^{4} \cdot 9 \cdot \left(\tfrac{2}{3}\right)^{k+4} = 9 \cdot \left(\frac{2}{3}\right)^4 \cdot \sum_{k=0}^{4} \cdot \left(\tfrac{2}{3}\right)^k$

$$\frac{16}{9} \cdot \frac{1-\left(\tfrac{2}{3}\right)^5}{1-\tfrac{2}{3}} = \frac{16}{9} \cdot 3 \cdot \frac{211}{243} = \frac{16 \cdot 211}{3^6} = \frac{3376}{729}$$

3) $\sum_{k=3}^{\infty} \frac{2^{2k} \cdot 2^1}{3 \cdot k!} = \tfrac{2}{3} \cdot \sum_{k=3}^{\infty} \frac{4^k}{k!}$

$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$

$$\tfrac{2}{3} \cdot \left[ \sum_{k=0}^{\infty} \frac{4^k}{k!} - \left( \frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} \right) \right]$$

$$\tfrac{2}{3} \cdot \left[ e^4 - (1 + 4 + 8) \right] = \tfrac{2}{3} \cdot \left(e^4 - 13\right)$$

4) $\sum (-3)^{k+1} \cdot \frac{2}{3^{4k}} = 2 \cdot \sum (-1)^{k+1} \cdot 3^{k+1} \cdot \frac{1}{3^{4k}}$

$6 \cdot \sum (-1)^{k+1} \cdot \frac{3^k}{3^{4k}} \Rightarrow \lim_{k\to\infty} \frac{3^k}{3^{4k}} = \lim_{k\to\infty} \left(\frac{1}{27}\right)^k = 0$ ✓

5) $\sum \frac{\sqrt{k} \cdot (k+2)^2}{(3k)^k} \Rightarrow \lim_{k\to\infty} \sqrt[k]{\frac{\sqrt{k} \cdot (k+2)^2}{(3k)^k}} = \left[\frac{1}{3k}\right] = 0$ ✓

$\sqrt[k]{\sqrt{k}} = \sqrt{\sqrt[k]{k}} = 1 \qquad \sqrt[k]{(k+2)^2} = \left(\sqrt[k]{k+2}\right)^2 = 1$

$\sqrt[k]{(3k)^k} = 3k$

6)

$\lim_{k\to\infty} \frac{(k+1)^2 \cdot 5^{k+1}}{(3\cdot(k+1))!} \frac{(3k)!}{k^2\ 5^k} = \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 \cdot \frac{5^{k+1}}{5^k} \cdot \frac{(3k)!}{(3k+3)!}$

$\underbrace{\left(1+\frac{1}{k}\right)^2}_{1} \cdot \underset{\downarrow}{5} \cdot \frac{\cancel{(3k)!}}{(3k+3)(3k+2)(3k+1)\cdot\cancel{(3k)!}}$

$\quad 5 \qquad 1/\infty \Rightarrow 0$ ✓