1) $f(x) = \dfrac{x^3 - 7x^2 + 2x + 40}{x^2 - x - 12}$

2) $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^3 + 5x^2 - 2x - 24}$

3) $f(x) = \dfrac{x^4 + 2x^3 - 5x^2 - 6x}{x^2 + 2x - 8}$

<u>Vieta:</u>

$$\begin{aligned} x^2 + px + q &= (x+a)\cdot(x+b) \\ &= x^2 + ax + bx + ab \\ &= x^2 + \underbrace{(a+b)}_{p}\cdot x + \underbrace{a\cdot b}_{q} \end{aligned}$$

$$f(x) = \frac{x^3 - 7x^2 + 2x + 40}{x^2 - x - 12}$$

$a \cdot b = -12$

$a + b = -1$

$(x-4)(x+3) \longrightarrow x_1 = 4 \vee x_2 = -3$

$$
\begin{aligned}
&(x^3 - 7x^2 + 2x + 40) : (x-4) = \underbrace{x^2 - 3x - 10} \\
&-(x^3 - 4x^2) \\
&\quad / \quad -3x^2 + 2x + 40 \\
&\quad -(-3x^2 + 12x) \\
&\qquad / \quad -10x + 40 \\
&\qquad -(-10x + 40) \\
&\qquad\quad / \quad /
\end{aligned}
$$

$a \cdot b = -10$

$a + b = -3$

$(x-5)(x+2)$

$$f(x) = \frac{(x-4)(x-5)(x+2)}{(x-4)(x+3)}$$

$$D_{f(x)} = \mathbb{R} \setminus \{-3; 4\}$$

$$\textcircled{4}$$

$$f_e(x) = \frac{(x-5)(x+2)}{x+3} = \frac{x^2-3x-10}{x+3} \quad ; D_{f_e(x)} = \mathbb{R} \setminus \{-3\}$$

$$\lim_{x \to 4} f(x) = f_e(4) = \frac{-1 \cdot 6}{7} = -6/7$$ => behebbare Lücke

$$\lim_{x \to -3^-} f(x) = \left[\frac{-8 \cdot (-1)}{0^-}\right] = -\infty$$

$$\lim_{x \to -3^+} f(x) = \left[\frac{8}{0^+}\right] = \infty$$

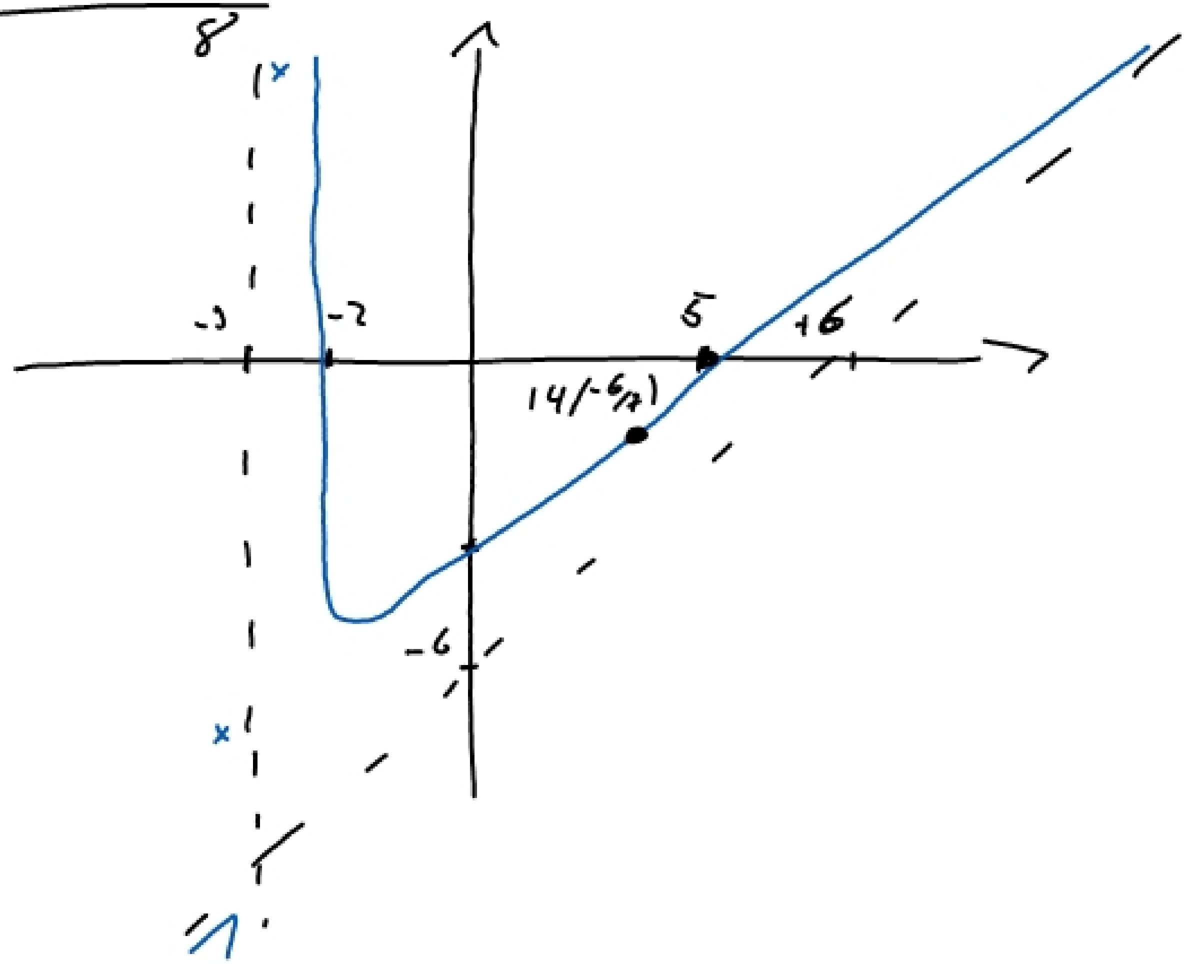
senkrechte Asymptote

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2(1 - \overbrace{3/x - 10/x^2}^{0})}{x\,(1 + \underbrace{3/x}_{0})} = [x] = \pm\infty$$

$$(x^2 - 3x - 10) : (x+3) = \underbrace{x - 6} + \frac{8}{x+3}$$

$$\begin{array}{l} -(x^2 + 3x) \\ \hline \quad -6x - 10 \\ -(-6x - 18) \\ \hline \qquad\qquad 8 \end{array}$$

$$d(x) = x - 6$$



2) $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^3 + 5x^2 - 2x - 24}$

$$(x^3 - 3x^2 + 4) : (x - 2) = \underbrace{x^2 - x - 2}$$

$$\begin{array}{l} -(x^3 - 2x^2) \\ \hline -\ -x^2 + 4 \\ -(-x^2 + 2x) \\ \hline -\ -2x + 4 \\ -(-2x + 4) \\ \hline -\ - \end{array}$$

$(x - 2)(x + 1)$

$$(x^3 + 5x^2 - 2x - 24) : (x - 2) = \underbrace{x^2 + 7x + 12}$$

$$\begin{array}{l} -(x^3 - 2x^2) \\ \hline -\ 7x^2 - 2x - 24 \\ -(7x^2 - 14x) \\ \hline -\ 12x - 24 \\ 12x - 24 \\ \hline -\ - \end{array}$$

$(x + 3)(x + 4)$

$$f(x) = \frac{(x-2)(x-2)(x+1)}{(x-2)(x+3)(x+4)} \quad ; \mathbb{D} = \mathbb{R} \setminus \{2; -3; -4\}$$

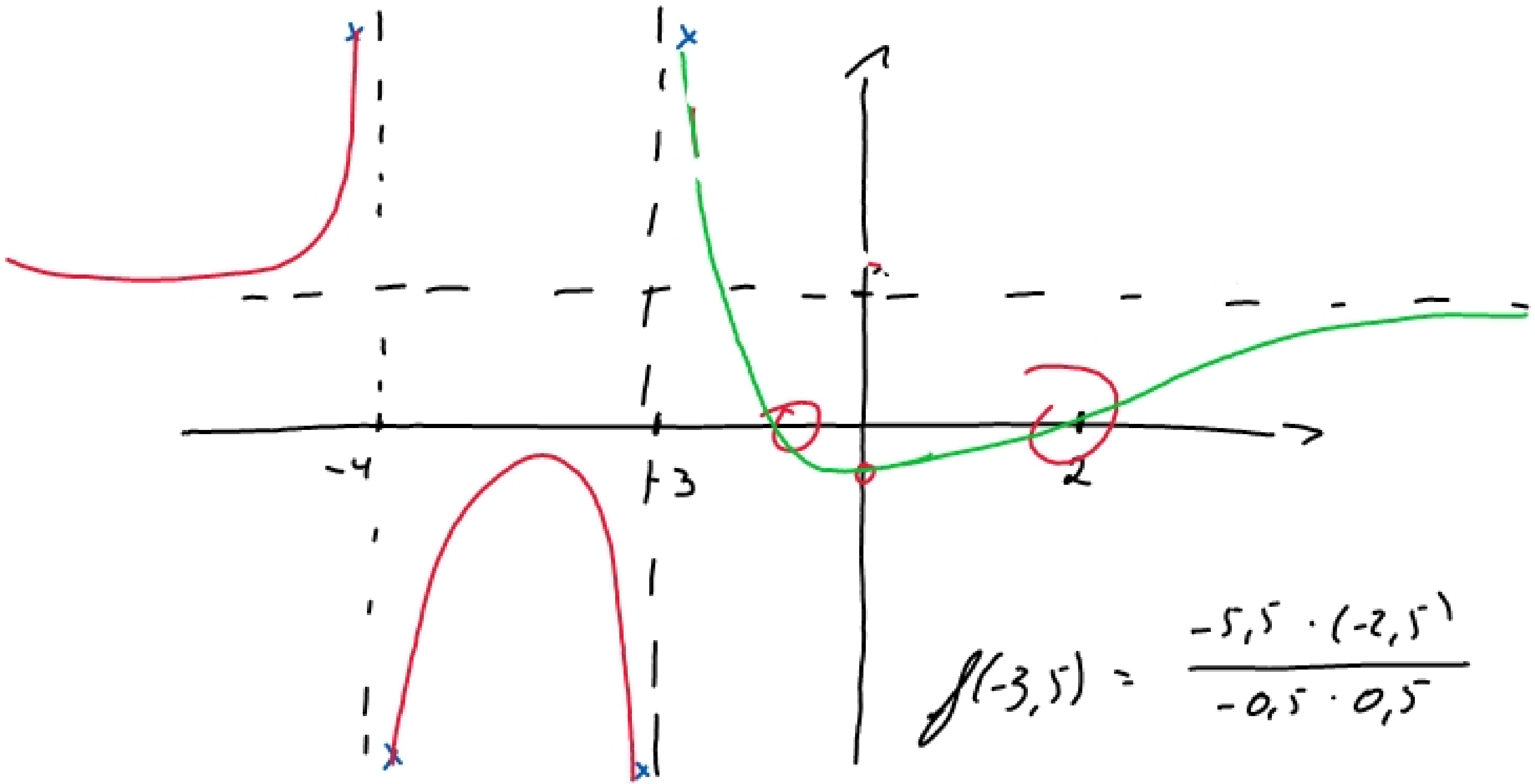
$$f_e(x) = \frac{(x-2)(x+1)}{(x+3)(x+4)} = \frac{x^2 - x - 2}{x^2 + 7x + 12} \quad ; \mathbb{D} = \mathbb{R} \setminus \{-3; -4\}$$

$$\lim_{x \to 2} f(x) = f_e(2) = 0 \quad \Rightarrow \text{S.L} \ (2/0)$$

$$\lim_{x \to \infty^{\pm}} f(x) = \lim_{x \to \pm\infty} \frac{x^2(1 - 1/x - 2/x^2)}{x^2(1 + 7/x + 12/x^2)} = 1 \stackrel{\wedge}{=} \text{u.A.}$$

$$\left.\begin{array}{l} \lim_{x \to -3^-} f_e(x) = \left[\frac{10}{1 \cdot 0^-}\right] = -\infty \Big/ \lim_{x \to -3^+} f_e(x) = \left[\frac{10}{1 \cdot 0^+}\right] = \infty \\ \lim_{x \to -4^-} f_e(x) = \left[\frac{18}{-1 \cdot 0^-}\right] = \infty \Big/ \lim_{x \to -4^+} f_e(x) = \left[\frac{18}{-1 \cdot 0^+}\right] = -\infty \end{array}\right\} \text{S.A.}$$

$$S_{x_1} = (2/0) \quad S_{x_2} = (-1/0) \quad S_y(0/-1/6)$$



$$f(-3{,}5) = \frac{-5{,}5 \cdot (-2{,}5)}{-0{,}5 \cdot 0{,}5}$$

$$f(-3{,}5) = -55$$