1) $\sum_{k=1}^{n}(\underline{3k-2}) = \frac{1}{2}n\cdot(3n-1)$

2) $1+4+7+\ldots+\overbrace{(3n-2)}^{a_n} = \frac{1}{2}n\cdot(3n-1)$

3) $\prod_{k=1}^{n}\left(3^{(2^k)}+1\right) = \frac{1}{8}\cdot\left[3^{(2^{n+1})}-1\right]$

4)* $\sum_{k=2}^{n}(k-1)\cdot\ln\left(\frac{k}{k-1}\right) = n\cdot\ln(n)-\ln(n!)$

4) $\sum_{k=2}^{n}\left(\frac{2k-3}{3^k}\right) = \frac{1}{3}-\frac{n}{3^n}$

1) $$\sum_{k=1}^{n} \underbrace{(3k-2)}_{a_k} = \underbrace{\tfrac{1}{2}\, n\,(3n-1)}_{S_n}$$

$n=1 \Rightarrow a_1 = S_1 \qquad 3\cdot 1 - 2 = 1 = \tfrac{1}{2}\cdot 1\cdot(3-1) = 1$ ✓

$n = n+1 \Rightarrow S_n + a_{n+1} = S_{n+1}$

$$\tfrac{1}{2}\cdot n\,(3n-1) + [3\cdot(n+1)-2] = \tfrac{1}{2}\cdot(n+1)\cdot[3\cdot(n+1)-1] \quad |\cdot 2$$

$$n\cdot(3n-1) + 2\cdot(3n+1) = (n+1)\cdot(3n+2)$$

$$3n^2 - n + 6n + 2 = 3n^2 + 3n + 2n + 2$$

$$3n^2 + 5n + 2 = 3n^2 + 5n + 2$$

$$0 = 0$$ ✓

3) $\prod_{k=1}^{n} \underbrace{\left[3^{(2^{k})}+1\right]}_{a_k} = \underbrace{1/8 \cdot \left[3^{(2^{n+1})}-1\right]}_{P_n}$

$n=1$ $\qquad a_1 = P_1 \qquad \left.\begin{array}{l} a_1 = 3^{2^1}+1 = 10 \\ P_1 = 1/8 \cdot \left[3^{2^2}-1\right] = 1/8 \cdot 80 = 10 \end{array}\right\} = \checkmark$

$n+1:$ $\qquad P_n \cdot a_{n+1} = P_{n+1}$

$$1/8 \cdot \left[3^{2^{n+1}}-1\right] \cdot \left[3^{2^{n+1}}+1\right] = 1/8 \cdot \left[3^{2^{n+2}}-1\right] \quad | \cdot 8$$

$$\left(3^{2^{n+1}}\right)^2 - 1 = 3^{2^{n+2}} - 1 \quad | +1$$

$$3^{2^1 \cdot 2^{n+1}} = 3^{2^{n+2}} = 3^{2^{n+2}}$$

$$0 = 0 \qquad \checkmark$$

4) $\displaystyle\sum_{k=2}^{n} \underbrace{\left(\frac{2k-3}{3^k}\right)}_{a_k} = \underbrace{1/3 - n/3^n}_{S_n}$

$n=2$: $\quad a_2 = S_2$

$$a_2 = \frac{2\cdot 2-3}{3^2} = 1/9$$

$$S_2 = 1/3 - 2/3^2 = 1/9$$

$$\left.\begin{matrix} a_2 = 1/9 \\ S_2 = 1/9 \end{matrix}\right\} =$$

$n+1$: $\quad S_n + a_{n+1} = S_{n+1}$

$$1/3 - n/3^n + \frac{2\cdot(n+1)-3}{3^{n+1}} = 1/3 - \frac{n+1}{3^{n+1}} \quad \Big| -1/3$$

$$-\frac{n}{3^n} + \frac{2n-1}{3^{n+1}} = -\frac{n+1}{3^{n+1}} \quad \Big| \cdot 3^{n+1} = 3\cdot 3^n$$

$$-3n + 2n - 1 = -(n+1)$$

$$0 = 0 \quad \checkmark$$