$$\left.\begin{array}{ll} 1) & \sum \dfrac{k!}{(2k+1)!} \left(x^2\right)^k \\[2em] 2) & \sum \dfrac{81^k}{k^3} \cdot x^{4k} \end{array}\right\} \text{max. Konvergenzbereich}$$

$$\left.\begin{array}{ll} 3) & f(x) = 2 \cdot \sin^4\left(\tfrac{1}{2}x + 2\pi\right) + 3 \\[1em] 4) & f(x) = 3 - 4 \cdot \cos^6\left(0{,}2 \cdot x + 5{,}5\pi\right) \end{array}\right\} \begin{array}{l} \text{Amplituden} \\ \text{Symmetrie} \\ \text{Periode} \end{array}$$

5) $f(x) = \Bigg\{$

1) $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} < 1$

$$\lim_{k \to \infty} \frac{\underline{(k+1)!} \quad \underline{(x^2)^{k+1}} \quad \underline{(2k+1)!}}{\underline{(2\cdot(k+1)+1)!} \quad \underline{k!} \quad \underline{(x^2)^k}}$$

$$\lim_{k \to \infty} \frac{(k+1)\cdot \cancel{k!}}{\cancel{k!}} \cdot \frac{\cancel{(2k+1)!}}{(2k+3)\cdot(2k+2)\cdot\cancel{(2k+1)!}} \cdot \frac{\cancel{(x^2)^k}\cdot(x^2)^1}{\cancel{(x^2)^k}}$$

$$\lim_{k \to \infty} \frac{\cancel{(k+1)} \quad x^2}{(2k+3)\cdot 2\cdot\cancel{(k+1)}} = \left[\frac{x^2}{4k+6}\right] \to \left[\frac{k}{\infty}\right] = 0 < 1$$

$\underline{\underline{x \in \mathbb{R}}}$ (Konvergenzbereich)

$$\lim_{k \to \infty} \frac{(k+1)\cdot x^2}{(2k+3)(2k+2)} \to \frac{(k+1)\cdot x^2}{4k^2+10k+6} = \frac{k\cdot(1+\frac{1}{k})\cdot x^2}{\underline{k^2}\cdot(4+\frac{10}{k}+\frac{6}{k^2})}$$

2) $\lim_{k \to \infty} \sqrt[k]{a_k} < 1$

$$\lim_{k \to \infty} \sqrt[k]{\frac{81^k}{k^3} \cdot x^{4k}} = \left| \frac{81}{1} \cdot x^4 \right| < 1 \quad \Rightarrow \quad |x^4| < 1/81$$

$x \in \left] -1/3 \, ; 1/3 \right[$

$x = -1/3 \qquad \underbrace{\sum \frac{81^k}{k^3} \cdot (-1/3)^{4k} \to \left[(-1/3)^4 \right]^k = (1/81)^k}_{\sum 1/k^3}$

$\sum 1/k^3 \qquad$ VGL: $\underline{\underline{1/k^3}} < 1/k^2$

$x = 1/3 \qquad \Rightarrow \sum 1/k^3$

$\Rightarrow \quad x \in \left[-1/3 \, ; 1/3 \right]$

3) $f(x) = 2 \cdot [\sin(\tfrac{1}{2}x + 2\pi)]^4 + 3$

$= 2 \cdot [\sin(\tfrac{1}{2}x) \cdot \underbrace{\cos(2\pi)}_{1} + \cos(\tfrac{1}{2}x) \cdot \underbrace{\sin(2\pi)}_{0}]^4 + 3$

$= 2 \cdot \sin^4(\tfrac{1}{2}x) + 3$

<u>W</u>: $2 \cdot [0; 1] + 3 = [0; 2] + 3 \Rightarrow y \in [3; 5]$

<u>Periode</u>: $P_{neu} = \frac{\pi}{1/2} = 2\pi \qquad f(x) = f(x + 2\pi)$

$2 \cdot \sin^4(\tfrac{1}{2}x) + 3 = 2 \cdot \sin^4(\tfrac{1}{2} \cdot (x + 2\pi)) + 3 \qquad | -3 \cdot \tfrac{1}{2}$

$\underline{\sin^4(\tfrac{1}{2}x)} = [\sin(\tfrac{1}{2}x + \pi)]^4$

$[\sin(\tfrac{1}{2}x) \cdot \underbrace{\cos(\pi)}_{-1} + \cos(\tfrac{1}{2}x) \cdot \underbrace{\sin(\pi)}_{0}]^4$

$[-\sin(\tfrac{1}{2}x)]^4 = \underline{\sin^4(\tfrac{1}{2}x)}$

Symmetrie: Achsen: $f(x) = f(-x)$

$$2 \cdot \sin^4(1/2\, x) + 3 = 2 \cdot \sin^4(-1/2\, x) + 3 \quad | -3 \cdot 1/2$$

$$\sin^4(1/2\, x) = [\sin(-1/2\, x)]^4$$

$$= [-\sin(1/2\, x)]^4$$

$$= \sin^4(1/2\, x)$$ ✓