

$$1) \quad \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{\cos(2x)}$$

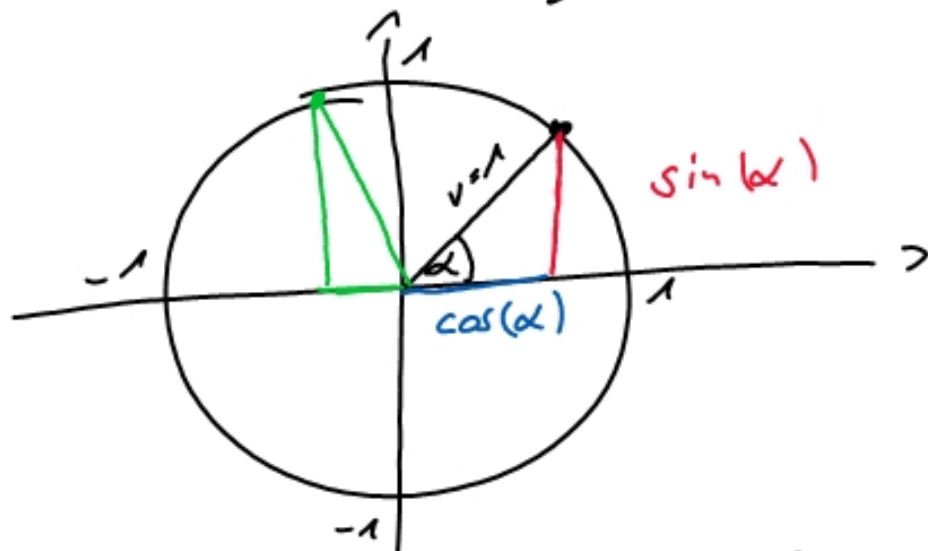
$$2) \quad \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x^2} - 1} \quad (2 \text{ Arten})$$

$$3) \quad \lim_{x \rightarrow 1} \frac{x^3 - 6x^2 + 9x - 4}{3 - 5x + x^2 + x^3} \quad (2 \text{ Arten})$$

$$4) \quad f(x) = \frac{x^3 - 2x^2 - x + 2}{3x^3 + 3x^2 - 3x - 3}$$

- > Faktorisierung
- > D / Ersatzfunktion
- > Interessante Grenzwerte

Trigonometrie

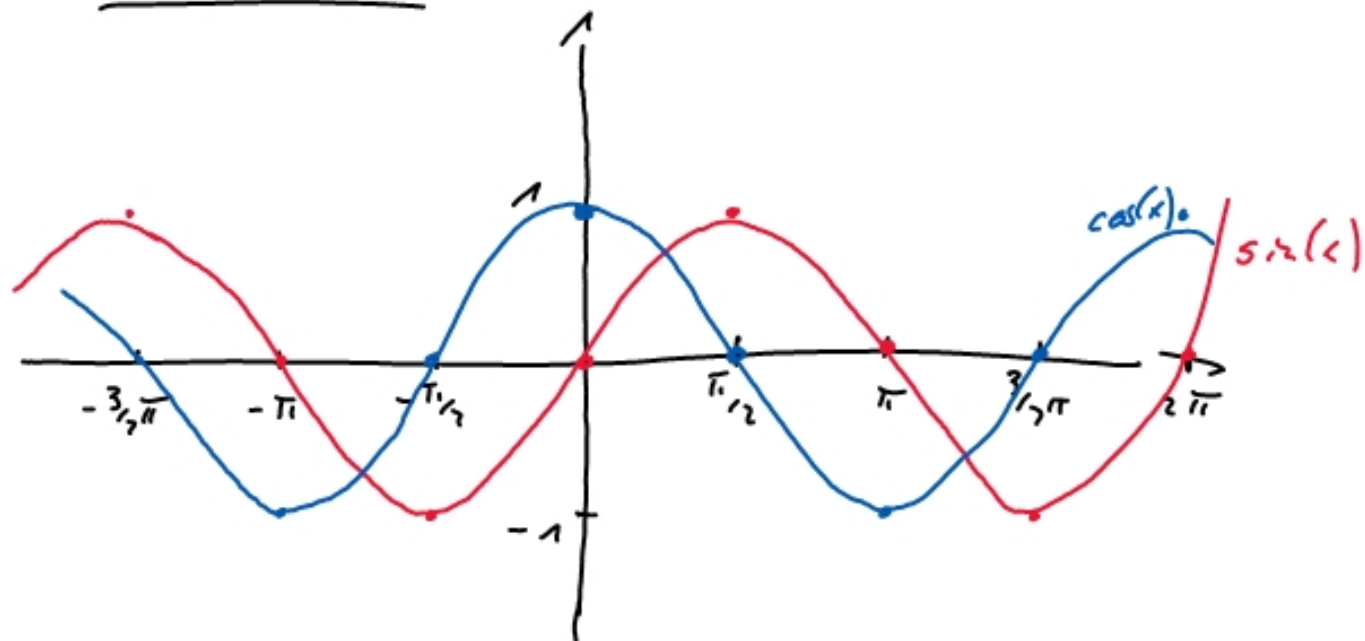


$$\lim_{x \rightarrow \pi/4} \frac{\sin(x) - \cos(x)}{\cos(2x)} = \left[\frac{0}{\cos(\pi/2)} \right] = \left[\frac{0}{0} \right]$$

$$\lim_{x \rightarrow \pi/4} \frac{\cos(x) - (-\sin(x))}{-2 \sin(2x)} = \frac{\cos(\pi/4) + \sin(\pi/4)}{-2 \cdot \sin(\pi/2)}$$

$$\rightarrow \frac{\sin(\pi/4) + \sin(\pi/4)}{-2 \cdot \sin(\pi/2)} = \frac{2 \cdot \sin(\pi/4)}{-2} = \overset{\lim}{-1/\sqrt{2}}$$

Funktion



$$2) \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1} = \frac{0}{0}$$

L'Hospital : $(\sqrt{1+3x})' = [(1+3x)^{1/2}]' = \frac{1}{2} \cdot (1+3x)^{-1/2} \cdot 3$

$$\lim_{x \rightarrow 0} \frac{1}{\frac{3}{2 \cdot \sqrt{1+3x}}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

\downarrow
 $\frac{3}{2 \sqrt{1+3x}}$

3. Binom

$$\frac{x}{\sqrt{1+3x} - 1} \cdot \frac{\sqrt{1+3x} + 1}{\sqrt{1+3x} + 1} = \frac{x \cdot (\sqrt{1+3x} + 1)}{\underbrace{1+3x}_{a^2} - \underbrace{1}_{b^2}}$$

$$\frac{\sqrt{1+3x} + 1}{3} = \frac{2}{3}$$

$$3) \lim_{x \rightarrow 1} \frac{x^3 - 6x^2 + 9x - 4}{3 - 5x + x^2 + x^3} = 0/0$$

L'Hospital: $\lim_{x \rightarrow 1} \frac{3x^2 - 12x + 9}{-5 + 2x + 3x^2} = 0/0$

$$\lim_{x \rightarrow 1} \frac{6x - 12}{2 + 6x} = \frac{-6}{8} = -\frac{3}{4}$$

$$\begin{array}{r}
 (x^3 - 6x^2 + 9x - 4)(x-1) = x^2 - 5x + 4 \\
 - (x^3 - x^2) \\
 \hline
 -5x^2 + 9x - 4 \\
 - (-5x^2 + 5x) \\
 \hline
 4x - 4 \\
 - (4x - 4) \\
 \hline
 0
 \end{array}$$

} Viete

$$\begin{array}{r}
 (x-1)(x-4) \\
 \hline
 (x-1)(x-1)(x+3)
 \end{array}$$