1) $a_{n+1} = a_n^2 + 1/4$; $a_0 = 0$; $n \geq 0$

(2) $a_{n+1} = \sqrt[3]{3a_n + 5}$; $a_1 = 1$; $n \geq 1$)

3) $a_{n+1} = 2 - \sqrt{2 - a_n}$; $a_1 = -1/4$, $n \geq 1$

4) $a_{n+1} = \left(\frac{a_n}{3}\right)^3 + 2$; $a_0 = 0$; $n \geq 0$

5) $3a_{n+1} = -2 \cdot (5 - a_n)$; $a_1 = 25$; $n \geq 1$

1)

$$\underbrace{a_{n+1} = a_n^2 + 1/4}_{\text{rekursive Folge}} \quad ; \quad \underbrace{a_0 = 0}_{\text{Startwert}} \quad ; \quad \underbrace{n \geq 0}_{\text{Laufvariable}}$$

$$a_0 = 0$$
$$a_1 = a_{0+1} = a_0^2 + 1/4$$
$$a_2 = a_{1+1} = a_1^2 + 1/4$$
$$\dots$$
$$a_{12} = a_{11+1} = a_{11}^2 + 1/4$$

$$a_n = n^2 + 1/4 \quad , \quad n \geq 0$$

$$a_0 = 0^2 + 1/4$$
$$a_1 = 1^2 + 1/4$$
$$a_{12} = 12^2 + 1/4$$

Monotonie: Tendenz $a_0 = 0 < a_1 = 1/4$

Behauptung $a_{n+1} > a_n$

$a_{n+1} > a_n$

$n = 0$ $\qquad a_1 > a_0 \qquad \frac{1}{4} > 0$ ✓

$n+1$ $\qquad a_{n+2} > a_{n+1}$

$$a_{n+1}^2 + \tfrac{1}{4} > a_n^2 + \tfrac{1}{4} \qquad \Big| -\tfrac{1}{4}$$

$$a_{n+1}^2 > a_n^2 \qquad \Big| \sqrt{}$$

$$a_{n+1} > a_n$$

<u>Schranken</u>: Da a_{n+1} streng monoton steigend ist, muss $a_0 = 0$ untere Schranke sein.

Behauptung: $a_n < 42$

$n = 0$ $\quad a_0 < 42 \quad 0 < 42$ ✓

<u>n+1</u>

$$\boxed{a_n < 42}$$

$$a_n^2 < 42^2 \quad | + 1/4$$

$$a_n^2 + 1/4 < 42^2 + 1/4$$

$$\rightsquigarrow$$

$$\Rightarrow a_{n+1} < 42^2 + 1/4$$

$$\left.\begin{array}{l} \lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} a_n \\ \lim\limits_{n\to\infty} a_n = \beta \end{array}\right\} \quad \begin{array}{l} \beta^2 + 1/4 = \beta \\ \beta^2 - \beta + 1/4 = (\beta - 1/2)^2 = 0 \\ \beta = 1/2 \end{array}$$

<u>Behauptung</u> $a_n < 1/2$

$n = 0$ $\quad a_0 = 0 < 1/2$ ✓

$n+1$

$$a_n < 1/2 \quad | \; (\;)^2$$

$$a_n^2 < 1/4 \quad | + 1/4$$

$$\underbrace{a_n^2 + 1/4}_{} < 1/4 + 1/4 = 1/2$$

$$a_{n+1} < 1/2$$

3) $a_{n+1} = 2 - \sqrt{2 - a_n}$; $a_1 = -1/4$; $n \geq 1$ ↑

$a_2 = 2 - \sqrt{2 - (-1/4)} = 2 - \sqrt{9/4} = 2 - 3/2 = 1/2$

<u>Behauptung</u>: $a_{n+1} > a_n$

$n = 1$ $\quad a_2 > a_1 \iff 1/2 > -1/4$ ✓

$n+1$ $\quad a_{n+2} > a_{n+1}$

$$2 - \sqrt{2 - a_{n+1}} > 2 - \sqrt{2 - a_n} \quad | -2 \cdot (-1)$$

$$\sqrt{2 - a_{n+1}} < \sqrt{2 - a_n} \quad | \, ()^2 - 2$$

$$-a_{n+1} < -a_n \quad | \, (-1)$$

$$a_{n+1} > a_n$$ ✓

Schranken: Da a_{n+1} streng monoton steigend ist, muss $a_1 = -14$ untere Schranke sein.

$$\lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} a_n$$

$$\lim_{n \to \infty} a_n = \beta$$

$$\left.\begin{array}{l} \\ \end{array}\right\} \quad 2 - \sqrt{2-\beta} = \beta \quad \Big| \begin{array}{l} +\sqrt{2-\beta} \\ -\beta \end{array}$$

$$2-\beta = \sqrt{2-\beta} \quad \big| \uparrow^2$$

$$(2-\beta)^2 = 2-\beta$$

$$4 - 4\beta + \beta^2 = 2 - \beta \quad \big| +\beta - 2$$

$$\beta^2 - 3\beta + 2 = 0$$

$$(\beta - 1)(\beta - 2) = 0$$

$$\boxed{\beta_1 = 1} \vee \beta_2 = 2$$

Schranke: Behauptung $a_n < 1$

$n = 1$ $\quad a_1 = -1/4 < 1$ ✓

n+1 $\quad a_n < 1 \quad | \cdot(-1) + 2$

$2 - a_n > 1 \quad | \sqrt{} \quad | \cdot(-1)$

$-\sqrt{2-a_n} < -1 \quad | +2$

$2 - \sqrt{2-a_n} < 1$

$a_{n+1} < 1$ ✓