

$$1) \quad a_{n+1} = a_n^2 + \frac{1}{4} ; a_0 = 0 ; n \geq 0$$

$$(2) \quad a_{n+1} = \sqrt[3]{3a_n + 5} ; a_1 = 1 ; n \geq 1$$

$$3) \quad a_{n+1} = 2 - \sqrt{2 - a_n} ; a_1 = -\frac{1}{4} ; n \geq 1$$

$$4) \quad a_{n+1} = \left(\frac{a_n}{3}\right)^3 + 2 ; a_0 = 0 ; n \geq 0$$

$$5) \quad 3a_{n+1} = -2 \cdot (5 - a_n) ; a_1 = 25 ; n \geq 1$$

1)

$$a_{n+1} = a_n^2 + 1/n \quad ; \quad a_0 = 0 \quad ; \quad n \geq 0$$

rekursive Folge

Startwert

Laufvariable

$$a_0 = 0$$

$$a_1 = a_{0+1} = a_0^2 + 1/n$$

$$a_2 = a_{1+1} = a_1^2 + 1/n$$

...

$$a_{12} = a_{11+1} = a_{11}^2 + 1/n$$

$$a_n = n^2 + 1/n, \quad n \geq 0$$

$$a_0 = 0^2 + 1/n$$

$$a_1 = 1^2 + 1/n$$

$$a_{12} = 12^2 + 1/n$$

Monotonie:

Tendenz

$$a_0 = 0 < a_1 = 1/n$$

Behauptung

$$a_{n+1} > a_n$$

Behauptung: $a_n < 42$

$n=0$

$$a_0 < 42$$

$$0 < 42$$

✓

$n+1$

$$a_n < 42$$

$$a_n^2 < 42^2$$

$$| + 1/4$$

$$a_n^2 + 1/4 < 42^2 + 1/4$$

✓

$$\rightarrow a_{n+1} < 42^2 + 1/4$$

$$\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n$$

$$\lim_{n \rightarrow \infty} a_n = \beta$$

$$\beta^2 + 1/4 = \beta$$

$$\beta^2 - \beta + 1/4 = (\beta - 1/2)^2 = 0$$

$$\beta = 1/2$$

Induktion

$$n = 0$$

$$n + 1$$

$$a_n < 1/2$$

$$a_0 = 0 < 1/2 \quad \checkmark$$

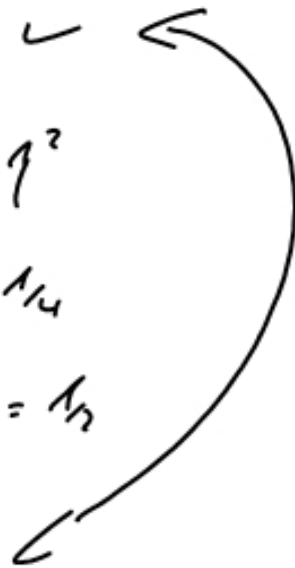
$$a_n < 1/2 \quad | \uparrow^2$$

$$a_n^2 < 1/4 \quad | + 1/4$$

$$a_n^2 + 1/4 < 1/4 + 1/4 = 1/2$$

$\underbrace{\hspace{2em}}$

$$a_{n+1} < 1/2$$



$$3) \quad a_{n+1} = 2 - \sqrt{2 - a_n} \quad ; \quad a_1 = -1/4 \quad ; \quad n \geq 1$$

$$a_2 = 2 - \sqrt{2 - (-1/4)} = 2 - \sqrt{9/4} = 2 - 3/2 = 1/2$$

Behauptung : $a_{n+1} > a_n$

$$n=1 \quad a_2 > a_1 \quad \Leftrightarrow \quad 1/2 > -1/4 \quad \checkmark$$

$$n+1 \quad a_{n+2} > a_{n+1}$$

$$2 - \sqrt{2 - a_{n+1}} > 2 - \sqrt{2 - a_n} \quad (-2 \cdot (-1))$$

$$\sqrt{2 - a_{n+1}} < \sqrt{2 - a_n} \quad (1^2 - 2)$$

$$-a_{n+1} < -a_n \quad (1 \cdot (-1))$$

$$a_{n+1} > a_n \quad \checkmark$$

Schranken:

Da a_{n+1} streng monoton steigend ist,
muss $a_1 = -1/4$ untere Schranke sein.

$$\left\{ \begin{array}{l} \lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n \end{array} \right.$$

$$\lim_{n \rightarrow \infty} a_n = \beta$$

$$2 - \sqrt{2-p} = \beta \quad \left| \begin{array}{l} + \sqrt{2-p} \\ - \beta \end{array} \right.$$

$$2-p = \sqrt{2-p} \quad | \uparrow^2$$

$$(2-p)^2 = 2-p$$

$$4 - 4p + p^2 = 2 - p \quad | +p - 2$$

$$p^2 - 3p + 2 = 0$$

$$(p-1)(p-2) = 0$$

$$\boxed{p_1 = 1} \vee p_2 = 2$$

Schritt: Behauptung $a_n < 1$

$$n=1 \quad a_1 = -1/4 < 1 \quad \checkmark$$

$$\begin{aligned} \underline{n+1} \quad a_n < 1 & \quad | \cdot (-1) + 2 \\ 2 - a_n > 1 & \quad | \sqrt{\quad} \quad | \cdot (-1) \\ -\sqrt{2 - a_n} < -1 & \quad | + 2 \end{aligned}$$

$$2 - \sqrt{2 - a_n} < 1$$

$$a_{n+1} < 1 \quad \checkmark$$