1) $\displaystyle \sum_{k=0}^{n} \underbrace{\left(\tfrac{1}{2}\right)^k}_{a_k} = \underbrace{2 - \tfrac{1}{2^n}}_{S_n}$

2) $\displaystyle \sum_{k=1}^{n} \frac{1}{k \cdot \sqrt{k}} \leq 3 - \frac{2}{\sqrt{n}}$

3) $\displaystyle \prod_{k=2}^{n} \left(1 - \tfrac{1}{k}\right) = \tfrac{1}{n}$

1) $n = 0$ $\quad a_0 = S_0 \iff \left(\tfrac{1}{2}\right)^0 = 2 - \tfrac{1}{2^0} \iff 1 = 2 - 1$ ✓

$n = n+1$ $\quad S_n + a_{n+1} = S_{n+1}$

$2 - \tfrac{1}{2^n} + \left(\tfrac{1}{2}\right)^{n+1} = 2 - \tfrac{1}{2^{n+1}} \quad | -2$

$-\frac{1}{2^n} + \frac{1}{2^{n+1}} = -\frac{1}{2^{n+1}} \quad | \cdot 2^{n+1} = 2^n \cdot 2^1$

$-2 + 1 = -1 \Rightarrow 0 = 0$ ✓

2) $n=1 \quad \frac{1}{1\cdot\sqrt{1}} \leq 3-\frac{2}{\sqrt{1}} \qquad 1\leq 1 \qquad \checkmark$

$$3-\frac{2}{\sqrt{n}}+\frac{1}{(n+1)(\sqrt{n+1})} \leq 3-\frac{2}{\sqrt{n+1}} \quad \Big| -3 \ \cdot \sqrt{n+1}$$

$$-\frac{2\cdot\sqrt{n+1}}{\sqrt{n}}+\frac{1}{n+1} \leq -2 \quad \Big| -\frac{1}{n+1}$$

$$-2\cdot\sqrt{\frac{n+1}{n}} \leq \frac{-2\cdot(n+1)}{n+1}-\frac{1}{n+1}$$

$$-2\cdot\sqrt{\frac{n+1}{n}} \leq \frac{-2n-3}{n+1} \quad \Big| :(-2)$$

$$\sqrt{\frac{n+1}{n}} \geq \frac{n+3/2}{n+1} \quad \Big| \ (\ )^2$$

$$\frac{n+1}{n} \geq \frac{(n+3/2)^2}{(n+1)^2} \quad \Big| \cdot(n+1)^2 \cdot n$$

$$(n+1)^3 \geq n\cdot(n+3/2)^2$$

$$\underline{n^3}+\underline{3n^2}+3n+1 \geq \underline{n^3}+\underline{3n^2}+9/4\,n \quad \Big| -1-9/4\,n$$

$$3/4\,n \geq -1 \qquad \checkmark$$

3) $\prod_{k=2}^{n} \left(1 - 1/k\right) = 1/n$

$n = 2$: $1 - 1/2 = 1/2$ ✓

$n \to n+1$ $P_n \cdot a_{n+1} = P_{n+1}$

$$\frac{1}{n} \cdot \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$$

$$\frac{1}{n} - \frac{1}{n\cdot(n+1)} = \frac{1}{n+1} \quad | \cdot n \cdot (n+1)$$

$$n+1 - 1 = n$$

$$0 = 0$$ ✓