S 153) 1) a)

$$\underbrace{\begin{matrix} 3 & 4 & -1 \\ 1 & -1 & 1 \\ 5 & 2 & 1 \end{matrix}}_{A} \left| \underbrace{\begin{matrix} 1 \\ 0 \\ 2 \end{matrix}}_{b} \right.$$

$$\det A: \left.\begin{matrix} -3 + 20 - 2 \\ - \\ 5 + 4 + 6 \end{matrix}\right\} 0$$

$$\Rightarrow \det A' = \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} = \begin{matrix} -3 \\ -4 \end{matrix} = -7 \neq 0 \Rightarrow Rg(A) = 2$$

$$\det(A|b) = \left.\begin{matrix} 8 - 1 \\ - \\ 2 + 2 \end{matrix}\right\} 3 \neq 0 \Rightarrow Rg(Ab) = 3$$

$\neq$

nicht lösbar

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 3 & 4 & -1 & 1 \\ 5 & 2 & 1 & 2 \end{array}\right) \quad |\cdot(-3) \; + \qquad |\cdot(-5) \; +$$

$$\left|\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 7 & -4 & 1 \\ 0 & 7 & -4 & 2 \end{array}\right| \quad |\cdot(-1) \; +$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 7 & -4 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$\det(A) = 0 \; ; \quad \det(A') = \begin{vmatrix} 1 & -1 \\ 0 & 7 \end{vmatrix} = 7 \neq 0 \Rightarrow Rg = 2$$

$$\det(A|b) = -\begin{vmatrix} 1 & -1 & 0 \\ 0 & 7 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -7 \neq 0 \Rightarrow Rg(A|b) = 3$$

S. 153 Nr. 2 5)

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 1 \\ 2 & 1 & 2 & -1 & 2 \\ 4 & 0 & 0 & 4 & 4 \\ -4 & -1 & -2 & 1 & -4 \end{array}\right) \quad \begin{array}{l} |\cdot(-1) \; + \quad |(-2) \; + \quad |\cdot 2 \; + \end{array}$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 \\ 0 & -2 & -6 & 4 & 2 \\ 0 & 1 & 4 & 1 & -2 \end{array}\right) \quad |\cdot(2) \; + \qquad \text{Pivot}$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 1 \\ 0 & 1 & 4 & 1 & -2 \\ 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & +2 & 6 & -2 \end{array}\right) \quad |\cdot 2 \; +$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 1 \\ 0 & 1 & 4 & 1 & -2 \\ 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right) \rightarrow \begin{array}{l} 4x_4 = 0 \\ x_4 = 0 \end{array}$$

$Det(A) = -8 \Rightarrow Rg(A) = 4 \mathrel{\hat{=}}$ Maximalrang

$det(A|S) = -8 \Rightarrow Rg(A|S) = 4$

$\Rightarrow$ LGS ist lösbar

2010

$$\left(\begin{array}{l} 2x + 3y - z = 2 - \beta \\ 3x + \alpha y + 4z = 5 \\ 7x + 4y + 2z = 8 \end{array}\right)$$

2011
$$\left(\begin{array}{lllll} a & + (x+1)\cdot b & + 0 & = & 1 \\ x\cdot a & + x^2\cdot b & - c & = & x-2 \\ 2a & + x\cdot b & - x\cdot c & = & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & x+1 & 0 & 1 \\ x & x^2 & -1 & x-2 \\ 2 & x & -x & -2 \end{array}\right)$$

$$
2010: \quad
\begin{array}{ccc|c}
z & x & y & \\
-1 & 2 & 3 & 2-\beta \\
4 & 3 & \alpha & 5 \\
2 & 7 & 4 & 8
\end{array}
\qquad I \cdot 4 \;+ \qquad I \cdot 2 \;+
$$

$$
\begin{array}{ccc|c}
-1 & 2 & 3 & 2-\beta \\
0 & 11 & 12+\alpha & 13-4\beta \\
0 & 11 & 10 & 12-2\beta
\end{array}
\qquad I\!I\,(-1) \;+
$$

$$
\left|\begin{array}{ccc|c}
-1 & 2 & 3 & 2-\beta \\
0 & 11 & 12+\alpha & 13-4\beta \\
0 & 0 & \boxed{-2-\alpha} & \boxed{-1+2\beta}
\end{array}\right|
$$

$0 \mid 0$: $\alpha = -2 \;\wedge\; \beta = 1/2$ (1 fach unendliche LSG)

$$
\left.\begin{array}{lll}
\det(A') = \begin{vmatrix} -1 & 2 \\ 0 & 11 \end{vmatrix} = -11 \neq 0 & & Rg(A) = 2 \\
\det(A|b) = \det(A') = -11 \neq 0 & & Rg(A|b) = 2
\end{array}\right\} =
$$

II. $0 \mid \mathbb{R}\setminus\{0\}$ $\alpha = -2 \wedge \beta \neq 1/2$

$$\det(A) = \begin{vmatrix} -1 & 2 \\ 0 & 11 \end{vmatrix} = -11 \neq 0 \quad \Rightarrow Rg(A) = 2$$

$$\det(A|b) = \begin{vmatrix} -1 & 2 & 2-\beta \\ 0 & 11 & 13-4\beta \\ 0 & 0 & -1+2\beta \end{vmatrix} = -11\cdot(2\beta-1) \neq 0$$

$$Rg(A|b) = 3$$

$\Big\}\ \neq$

$\Rightarrow$ nicht lösbar

III. $\mathbb{R}\setminus\{0\} \mid \mathbb{R}$ $\alpha \neq -2$ (eindeutig lösbar $\wedge$ 1 Lösung)

$$\det(A) = \begin{vmatrix} -1 & 2 & 3 \\ 0 & 11 & 12+\alpha \\ 0 & 0 & -2-\alpha \end{vmatrix} = -11(-2-\alpha) \neq 0$$

$Rg(A) = 3$ (Maximalrang)

$\Big\}\ =$

$$\det(A|b) = -11(-2-\alpha) \neq 0 \quad \Rightarrow Rg(A|b) = 3$$

5) $\alpha = -2 \wedge \beta = 1/2$

$$\left|\begin{array}{ccc|c} 2 & 3 & -1 & 3/2 \\ 11 & 10 & 0 & 11 \\ 0 & 0 & 0 & 0 \end{array}\right| \qquad x_1 = \gamma$$

$$2\gamma + 3x_2 - x_3 = 3/2$$

$$11\gamma + 10x_2 = 11 \qquad x_2 = \frac{11}{10} - \frac{11}{10}\gamma$$

$$2\gamma + \frac{33}{10} - \frac{33}{10}\gamma - x_3 = 3/2$$

$$x_3 = \frac{33}{10} - \frac{15}{10} + \gamma\left(\frac{20}{10} - \frac{33}{10}\right) = \frac{18}{10} - \frac{13}{10}\gamma$$

$$\vec{x} = \begin{pmatrix} \gamma \\ \frac{11}{10} - \frac{11}{10}\gamma \\ \frac{18}{10} - \frac{13}{10}\gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 1{,}1 \\ 1{,}8 \end{pmatrix} + \gamma \begin{pmatrix} 10 \\ -11 \\ -13 \end{pmatrix}$$

$$2011: \left(\begin{array}{ccc|c} 1 & x+1 & 0 & 1 \\ x & x^2 & -1 & x-2 \\ 2 & x & -x & -2 \end{array}\right) \quad |(-x) \; |(-2)$$

$$\left(\begin{array}{ccc|c} 1 & x+1 & 0 & 1 \\ 0 & -x & -1 & -2 \\ 0 & -x-2 & -x & -4 \end{array}\right) \quad |(-x)$$

$$\begin{array}{ccc|c} 1 & x+1 & 0 & 1 \\ 0 & -x & -1 & -2 \\ 0 & x^2-x-2 & 0 & 2x-4 \end{array}$$

$$(x-2)(x+1) \;\big|\; 2\cdot(x-2)$$

$0 | 0 \Rightarrow x = 2$

∞ LSG

$\mathbb{R}\setminus\{0\} \mid \mathbb{R} \Rightarrow x \in \mathbb{R}\setminus\{-1;2\}$

1 LSG

$0 \mid \mathbb{R}\setminus\{0\}$

$x = -1 \wedge$

$x \neq 2$

kein LSG

WS 2010/11

siehe Skript

$$\left.\begin{array}{rcl} x + 2y + 3z &=& 2 \\ -x - y - 2z &=& 1 \\ 3x + y + \alpha \cdot z &=& \beta \end{array}\right\} \begin{array}{l} 1 \text{ LSG} \\ \text{kein LSG} \\ \infty \text{ LSG} \Rightarrow \text{Gerade} \end{array}$$

$$\left(\begin{array}{rcl} x_1 - 2x_2 + 3x_3 &=& -4 \\ 2x_1 + x_2 + x_3 &=& 2 \\ x_1 + \alpha x_2 + 2x_3 &=& -\beta \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 2 & 1 & 1 & 2 \\ 1 & \alpha & 2 & -\beta \end{array}\right) \quad |\,(-2)\ + \quad |\,(-1)\ +$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 5 & -5 & 10 \\ 0 & \alpha+2 & -1 & 4-\beta \end{array}\right) \quad |\,(-1/5)\ +$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & -1 & 1 & -2 \\ 0 & \alpha+1 & 0 & 2-\beta \end{array}\right)$$

∞ Lösungen : $\alpha = -1 \wedge \beta = 2$

1 Lösung : $\alpha \neq -1$

keine Lösung : $\alpha = -1 \wedge \beta \neq 2$

∞ Lösungen: $\alpha = -1 \wedge \beta = 2$

$$\det(A') = \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = -1 \neq 0 \Rightarrow Rg(A) = 2$$

$$\det(A|b) = \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = -1 \neq 0 \Rightarrow R(A|b) = 2$$

$=$

1. Lösung: $\alpha \neq -1$

$$\det(A) = \begin{vmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & \alpha+1 \end{vmatrix} = (\alpha+1) \neq 0 \Rightarrow Rg(A) = 3$$

$$\det(A|b) = (\alpha+1) \neq 0 \Rightarrow Rg(A|b) = 3$$

Maximalrang

$=$

Keine Lösung: $\alpha = -1 \wedge \beta \neq 2$

$$Det(A') = \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = -1 \neq 0 \Rightarrow Rg(A) = 2$$

$\neq$

$$\det(A|b) = \begin{vmatrix} 1 & -2 & -4 \\ 0 & -1 & -2 \\ 0 & 0 & 2-\beta \end{vmatrix} = (\beta - 2) \neq 0 \Rightarrow Rg(A|b) = 3$$

Geraden:

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad x_2 = r$$

$$\begin{aligned} 1x_1 - 2r + 3x_3 &= -4 \\ -r + x_3 &= -2 \qquad \Rightarrow x_3 = r - 2 \end{aligned}$$

$$\begin{aligned} x_1 - 2r + 3(r-2) &= -4 \\ x_1 &= -r + 2 = 2 - r \end{aligned}$$

$$\vec{x} = \begin{pmatrix} 2 - r \\ 0 + r \\ r - 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} + r \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad (; r \in \mathbb{R})$$