

$$\begin{vmatrix} 2 & 1 & -1 & 3 \\ 4 & 2 & 3 & 4 \\ 1 & -1 & 2 & 3 \\ 2 & -4 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 4 \\ 1 & -1 & 1 & 3 \\ 2 & -4 & -1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 4 \\ 1 & -1 & 1 & 3 \\ 3 & -5 & 0 & 4 \end{vmatrix} = \underbrace{0 \cdot A_{13}} + 5 \cdot A_{23} + 1 \cdot A_{33} + \underbrace{0 \cdot A_{34}}$$

$$= 5 \cdot (-1)^5 \cdot D_{23} + 1 \cdot (-1)^6 \cdot D_{33}$$

$$-5 \cdot \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 3 \\ 3 & -5 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 4 \\ 3 & -5 & 4 \end{vmatrix}$$

$$-5 \cdot \begin{pmatrix} -8 + 9 - 11 \\ -9 + 4 - 30 \end{pmatrix} + \begin{pmatrix} 18 + 12 - 60 \\ 18 + 16 - 40 \end{pmatrix} = \begin{pmatrix} -5 \cdot (-14 + 35) \\ + \\ (-32 + 6) \end{pmatrix}$$

$$-105 - 26 = -131$$

$$\left| \begin{array}{cccc|c} 2 & 3 & 1 & -4 & \\ 1 & 2 & 3 & -1 & \\ 4 & -3 & 2 & 1 & \\ 1 & 2 & -1 & 3 & \end{array} \right| \xrightarrow{+}$$

$$\left| \begin{array}{cccc|c} 2 & 3 & 1 & -4 & \\ 1 & 2 & 3 & -1 & \\ -6 & -0 & -3 & -3 & \\ 1 & 2 & -1 & 3 & \end{array} \right|$$

$$\left| \begin{array}{cccc|c} 2 & 3 & 1 & -3 & \\ 1 & 2 & 3 & 2 & \\ 6 & 0 & 3 & 0 & \\ 1 & 2 & -1 & 2 & \end{array} \right| \xrightarrow{-\frac{1}{2}}$$

$$\left| \begin{array}{cccc|c} 2 & 3 & 2 & -3 & \\ 1 & 2 & 6 & 2 & \\ 6 & 0 & -6 & 0 & \\ 1 & 2 & -2 & 2 & \end{array} \right|$$

$$-\frac{1}{2} \cdot \left| \begin{array}{cccc|c} 2 & 3 & 4 & -3 & \\ 1 & 2 & 7 & 2 & \\ 6 & 0 & 0 & 0 & \\ 1 & 2 & -1 & 2 & \end{array} \right| \leftarrow -\frac{1}{2} \cdot (6) \cdot (-1)^4$$

$$\left| \begin{array}{ccc|c} 3 & 4 & -3 & 42 + 16 + 6 \\ 2 & 7 & 2 & - \\ 2 & -1 & 2 & -42 + 16 - 6 \end{array} \right|$$

$$\boxed{-3 \cdot 96 = -288}$$