$$\vec{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 7 \\ -2 \\ -3 \end{pmatrix}$$

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{7^2 + (-2)^2 + (-3)^2} = \sqrt{62}$$

$$\varphi(\vec{a}, \vec{b}) = 2 \cdot 7 + 3 \cdot (-2) + 1 \cdot (-3) = 5$$ → Skalarprodukt

$$\alpha = \arccos \frac{5}{\sqrt{14} \cdot \sqrt{62}}$$

Vektorprodukt

$$\vec{a} \times \vec{b} = \begin{pmatrix} 3 \cdot (-2) - (-2) \cdot 1 \\ 1 \cdot 7 - (-3) \cdot 2 \\ 2 \cdot (-2) - 3 \cdot 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 13 \\ -25 \end{pmatrix}$$

$$\underbrace{\vec{a} \;;\; \vec{b}}_{\text{linear abhängig}} \;;\; \vec{c}$$

$$\gamma \cdot \vec{a} = \vec{b}$$
$$= \beta \cdot \vec{b}$$

$$\alpha \cdot \vec{a} + \beta \cdot \vec{b} = \vec{c}$$

$$\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$$

=> Linearkombination

$$\alpha = \beta = \gamma = 0$$ Triviallösung

transponiert

3) a) $\vec{a} = (-2; 3; 1)^T$; $\vec{b} = (1; 2; 2)^T$

$$g: \vec{x} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} + \alpha \cdot \begin{pmatrix} 1 + 2 \\ 2 - 3 \\ 2 - 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -2 - 1 \\ 3 - 2 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

4)

$$\vec{x} = \overset{\vec{a}}{\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}} + \gamma \overset{\vec{b}}{\begin{pmatrix} 2 + 1 \\ -1 - 4 \\ 2 - 3 \end{pmatrix}} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} + \gamma \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$$

Punktprobe:

$$\begin{pmatrix} 5 \\ -6 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} + \gamma \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 6 \\ -10 \\ -2 \end{pmatrix} = \gamma \cdot \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$$

alle Punkte liegen auf einer Geraden ← $\gamma = 2$

1) a) $g_1: \vec{x}_1 = \begin{pmatrix}2\\-5\\1\end{pmatrix} + \alpha\begin{pmatrix}-2\\8\\-4\end{pmatrix}$; $g_2: \vec{x}_2 = \begin{pmatrix}-1\\3\\-2\end{pmatrix} + \beta\begin{pmatrix}1\\-4\\2\end{pmatrix}$

I Richtungsvektoren:

$$\begin{pmatrix}-2\\8\\-4\end{pmatrix} + \gamma \cdot \begin{pmatrix}1\\-4\\2\end{pmatrix} \Rightarrow \begin{matrix}\gamma = -2\\ \gamma = -2\\ \gamma = -2\end{matrix} \quad \checkmark$$

II Punktprobe:

$$\begin{pmatrix}-1\\3\\-2\end{pmatrix} = \begin{pmatrix}2\\-5\\1\end{pmatrix} + \alpha\begin{pmatrix}-2\\8\\-4\end{pmatrix} \quad \Bigg| - \begin{pmatrix}2\\-5\\1\end{pmatrix}$$

$$\begin{pmatrix}-3\\8\\-3\end{pmatrix} = \alpha \cdot \begin{pmatrix}-2\\8\\-4\end{pmatrix} \qquad \left.\begin{matrix}\alpha = 3/2\\ \alpha = 1\end{matrix}\right\} \lightning$$

$\Rightarrow$ **parallel**

2)

a) $g_1: \vec{x}_1 = \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} + \alpha \begin{pmatrix} 4 & + & 1 \\ -1 & + & 4 \\ 2 & - & 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} + \alpha \underline{\begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}}$

$g_2: \vec{x}_2 = \begin{pmatrix} 9 \\ 2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -6 & - & 9 \\ 7 & - & 2 \\ 4 & - & 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 1 \end{pmatrix} + \beta \underline{\begin{pmatrix} -15 \\ 5 \\ 3 \end{pmatrix}}$

I $\quad \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} = \alpha \begin{pmatrix} -15 \\ 5 \\ 3 \end{pmatrix} \qquad \begin{matrix} \alpha = -1/3 \\ \alpha = 3/5 \end{matrix} \qquad ↯$

II $\quad g_1 = g_2: \quad \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix} + \alpha \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -15 \\ 5 \\ 3 \end{pmatrix}$

$5\alpha + 15\beta = 10$

$3\alpha - 5\beta = 6$

$-\alpha - 3\beta = -2 \Rightarrow \alpha = -2 + 3\beta$

$$\left| \begin{array}{l} 5 \cdot (3p-2) + 15p = 10 \\ 3 \cdot (3p-2) - 5p = 6 \end{array} \right|$$

$$15p - 10 + 15p = 10$$
$$30p = 20$$
$$p = 2/3$$

$$9p - 6 - 5p = 6$$
$$14p = 12$$
$$p = 6/7$$

↯ $\Rightarrow$ windschief ✓