
Mathe

3) $(x \cup (\overline{y \wedge x})) \wedge ((z \cup x) \cup (\bar{x} \cup z))$

Assoz. komm.

de Morgan $(x \cup (\bar{y} \cup \bar{x})) \wedge (z \cup (x \cup \bar{x}))$

$(\bar{y} \cup (x \cup \bar{x}))$

Tertium non datur

$(\bar{y} \cup w) \wedge (z \cup w)$

$\Omega \qquad \Omega$

übergreift

$w \wedge w$

$\Omega \wedge \Omega$

$w \quad \Omega$

4) $X = \{0; 1\}$

a) $P(X) = \{ \{\}, \{0\}; \{1\}; \{0;1\} \}$

b) $R = \{ (a;b) \in P(X) \times P(X) \mid a \subseteq b \}$

$\{ (\{\}, \{\}); (\{\}, \{0\}); (\{\}; \{1\}); (\{\}, \{0;1\});$

$(\{0\}; \{0\}); (\{0\}; \{0;1\}); (\{1\}; \{1\});$

$(\{1\}; \{0;1\}); (\{0;1\}; \{0;1\}) \}$

5) $(r \lor (p \to q)) \land (\neg r \lor q)$

KNF: $(r \lor \neg p \lor q) \land (\neg r \lor q)$

DNF: $(r \land \neg r) \lor (\neg p \land \neg r) \lor (q \land \neg r) \lor (r \land q) \lor (\neg p \lor q)$

$\quad F \quad \lor \qquad\qquad\qquad\qquad\qquad\qquad \underbrace{\lor (q \lor q)}_{\lor\, q}$

7) $R = \{(a;b) \in \mathbb{N}\times\mathbb{N} \mid \sqrt{a}-\sqrt{b} = 2\cdot k;\ k\in\mathbb{N}\}$

<u>reflexiv:</u> $(a,a)\in R \quad ; a\in\mathbb{N}$

$$\sqrt{a}-\sqrt{a} = 0 = 2\cdot k \quad ; 0\in\mathbb{N}$$

<u>transitiv:</u>

$$(a,b)\in R \Rightarrow \sqrt{a}-\sqrt{b} = 2\cdot k_1$$
$$(b,c)\in R \Rightarrow \sqrt{b}-\sqrt{c} = 2\cdot k_2$$
$$(a,c)\in R \Rightarrow \sqrt{a}-\sqrt{c} = 2\cdot k_3$$

$$\sqrt{b} = \sqrt{a} - 2k_1$$
$$\sqrt{a} - 2k_1 - \sqrt{c} = 2k_2 \quad |+2k_1$$
$$\sqrt{a}-\sqrt{c} = 2k_2 + 2k_1 = 2\cdot(\underbrace{k_1+k_2}_{k_3})$$

Symmetrie : $(16,4) \in R \quad \sqrt{16} - \sqrt{4} = 2 = 2 \cdot 1$ ✓

$(4,16) \notin R \quad \sqrt{4} - \sqrt{16} = -2 = 2 \cdot (-1)$ ↯

$$(a,b) \in R \wedge (b,a) \notin R \quad , \quad a \neq b \; ; k \neq 0$$

$$\sqrt{a} - \sqrt{b} = 2 \cdot k_1 \qquad \sqrt{b} - \sqrt{a} = 2 \cdot k_2$$

$$\sqrt{b} = \sqrt{a} - 2k_1 \qquad \sqrt{a} - 2k_1 - \sqrt{a} = 2k_2$$

$$2k_1 + 2k_2 = 2 \cdot \underbrace{(k_1 + k_2)}_{k_1 = k_2 = 0} = 0$$

$\Rightarrow$ **Ordnungsrelation**

9) a) $f(x) = \sqrt{13-3x}$; $\mathbb{D}_f = \{x \in \mathbb{R} \mid x \leq 13/3\}$

$g(x) = x-1$; $\mathbb{D}_g = \mathbb{R}$

$f(x) = g(x)$

$\sqrt{13-3x} = x-1 \quad | ()^2$

$13-3x = (x-1)^2 = x^2-2x+1$

$x^2+x-12 = 0 \qquad x_1 = 3 \lor x_2 = -4$

$(x-3)(x+4) = 0$

$g(3) = 2 \quad \Rightarrow S_1(3/2)$

$g(-4) = -5 \quad \Rightarrow S_2(-4/-5)$

5) $\frac{2x^{\cancel{2}}}{\cancel{x}(x+4)} - \frac{5x+4}{3\cdot(x+4)} = \frac{x-5}{3x}$ $\quad D = \mathbb{R} \setminus \{-4; 0\}$

$| \cdot 3 \cdot x \cdot (x+4)$

$2x \cdot 3 \cdot x - (5x+4) \cdot x = (x-5) \cdot (x+4)$

$6x^2 - 5x^2 - 4x = x^2 - 5x + 4x - 20$

$x^2 - 4x = x^2 - x - 20 \quad | + x - x^2$

$-3x = -20$

$x = 20/3$

$\cancel{f(-4) = \frac{-9}{-12} = 3/4}$ $\quad \cancel{S(-4 \mid 3/4)}$

$f\left(\frac{20}{3}\right) = \beta$ $\quad S\left(\frac{20}{3} \mid \beta\right)$

11) $9^n + 7 = 8k$; $k \in \mathbb{N}$

$n = 0$ $\quad 9^0 + \cancel{7} = 8 = 8 \cdot 1$; $1 \in \mathbb{N}$ ✓

Prämisse $\boxed{9^n + 7}$ ist für $n \in \mathbb{N}$ durch 8 teilbar.

$n+1$: $\quad 9^{n+1} + 7 = 9 \cdot 9^n + 7$

$$\underbrace{9^n + 7} + 8 \cdot 9^n$$

$$8 \cdot k_1 + 8 \cdot n = 8 \cdot (\underbrace{k_1 + n}_{\mathbb{N} + \mathbb{N}}) = 8 \cdot \underset{\mathbb{N}}{\underset{\downarrow}{k_3}}$$

1) $1 + n \cdot a \leq (1 + a)^n \quad ; n \in \mathbb{N}$

$n = 0 : \quad 1 + 0 \leq (1 + a)^0 = 1$ ✓

Es gilt $1 + n \cdot a \leq (1 + a)^n \quad ; n \in \mathbb{N}$

$n + 1 : \quad 1 + (n+1) \cdot a \leq (1 + a)^{n+1}$

$1 + n \cdot a + a \leq (1 + a)^n \cdot (1 + a)^1$

$1 + n \cdot a + a \leq (1 + n \cdot a)(1 + a)$

$1 + n \cdot a + a + n \cdot a^2 \quad | -1 - n \cdot a - a$

$n \cdot a^2 \geq 0$

$n \in \mathbb{N} \cdot \geq 0$

14) $a_n = \sqrt{2} + e^{-2n}$

$\Delta: \quad a_{n+1} - a_n = \sqrt{2} + e^{-2(n+1)} - \left(\sqrt{2} + e^{-2n}\right)$

$= e^{-2n-2} - e^{-2n} = \underset{>0}{e^{-2n}} \cdot \underbrace{\left(e^{-2} - 1\right)}_{\frac{1}{e^2} - 1 < 0}$

$\Rightarrow a_n$ ist fallend

$a_1 = \sqrt{2} + \frac{1}{e^2}$ ist obere Schranke

$a_n > \sqrt{2}$ $\quad n = 1 \quad a_1 = \sqrt{2} + \frac{1}{e^2} > \sqrt{2}$ ✓

$n+1 \quad a_{n+1} > \sqrt{2}$

$\sqrt{2} + e^{-2(n+1)} > \sqrt{2} \quad | -\sqrt{2}$

$e^{-2n-2} > 0$ ✓

$a_{n+1} = \sqrt{a_n} + 2 \quad ; \quad a_1 = 2{,}25 = \frac{9}{4} \qquad n \geq 1$

$a_2 = \sqrt{\frac{9}{4}} + 2 = \frac{3}{2} + 1 = 3{,}5$

Behauptung: $\quad a_{n+1} > a_n$

$n = 1 \qquad a_2 = 3{,}5 > 2{,}25 = a_1 \qquad \checkmark$

$n+1 \qquad a_{n+2} > a_{n+1}$

$\sqrt{a_{n+1}} + 2 > \sqrt{a_n} + 2 \quad | - 2$

$\sqrt{a_{n+1}} > \sqrt{a_n} \quad | ()^2$

$a_{n+1} > a_n \qquad \checkmark$

Schranke $\quad a_1 = 2{,}25$ ist untere Schranke

$a_n < 4 \quad \rightarrow \quad$ IA: $\; a_1 = 2{,}25 < 4 \; ✓$

$a_n < 4 \quad | \sqrt{\;}$

$\sqrt{a_n} < \sqrt{4} = 2 \quad | +2$

$\underbrace{\sqrt{a_n} + 2}_{a_{n+1}} < 4$

$a_{n+1} < 4 \; ✓$

Grenzwert: $\lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} a_n$

$\lim\limits_{n\to\infty} = \gamma$

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$$\gamma = \sqrt{\gamma} + 2 \quad | -2$$
$$\gamma - 2 = \sqrt{\gamma} \quad | \uparrow^2$$
$$(\gamma - 2)^2 = \gamma$$
$$\gamma^2 - 4\gamma + 4 = \gamma$$
$$\gamma^2 - 5\gamma + 4 = 0$$
$$(\gamma - 4)(\gamma - 1) = 0$$

16) $\sum_{k=2}^{\infty} (-1)^{k+1} \cdot \frac{3\,9^{k}}{(2k+1)!}$

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(-1)^{(k+1)+1} \cdot 3 \cdot 9^{k+1} \quad (2k+1)!}{(2\cdot(k+1)+1)! \quad (-1)^{k+1}\, 3 \cdot 9^{k}}$$

$$\frac{(-1)^{k+2}}{(-1)^{k+1}} \cdot \frac{3 \cdot 9^{k+1}}{3 \cdot 9^{k}} \cdot \frac{(2k+1)!}{(2k+3)!}$$

$$\underbrace{(-1) \cdot \quad 9^{1} \cdot \frac{1 \quad (2k+1)!}{(2k+3)(2k+2)\ (2k+1)!}}$$

$$-1 \cdot 9 \cdot \frac{1}{\infty} \to 0 < 1 \quad ✓$$

$$\sum_{k=2}^{\infty} (-1)^{k+1} \cdot \frac{3 \cdot 9^k}{(2k+1)!} = -1 \cdot \underbrace{\sum_{k=2}^{\infty} (-1)^k \cdot \frac{3^{2k+1}}{(2k+1)!}}_{\sin(3)}$$

$$- \left[\sin(3) - \left[\frac{3^1}{1!} + (-1) \cdot \frac{27}{6} \right] \right]$$

$$- \sin(3) + \left(3 - 9/2 \right) = - \sin(3) - 3/2$$

17) $\sum \frac{k^n \cdot x^k}{\sqrt{42 \cdot k!}}$; $n \in \mathbb{N}$

$$\frac{(k+1)^n}{k^n} = \left(\frac{k+1}{k} \right)^n = \left(\frac{k}{k} + \frac{1}{k} \right)^n$$

$$\lim_{k \to \infty} \frac{(k+1)^n \cdot x^{k+1} \sqrt{42 \cdot k!}}{\sqrt{42 \cdot (k+1)!} \; k^n \cdot x^k}$$

$$\frac{(k+1)^n}{k^n} \cdot \frac{x^{k+1}}{x^k} \cdot \frac{\sqrt{42} \cdot \sqrt{k!}}{\sqrt{42} \cdot \sqrt{(k+1)\, k!}}$$

$$\underline{\left(1 + \frac{1}{k} \right)^n} \cdot x \cdot \frac{1}{\sqrt{k+1}} \to 1 \cdot x \cdot \frac{1}{\infty} \to 0$$

19) a) $\lim\limits_{x \to \infty} \left[\frac{3}{x} - \left(1 + \frac{3}{x}\right)^x \right] = \frac{3}{\infty} - e^3 = -e^3$

$\lim\limits_{x \to \infty} \left(1 + \frac{r}{x}\right)^x = e^r$

b) $\lim\limits_{x \to 2} \frac{\cos(5\pi - x \cdot \pi)}{4x - 8} = \left[\frac{\cos(3\pi)}{0} \right] = \frac{-1}{0} = -\infty$

$\lim\limits_{x \to 2} \frac{\sin(5\pi - x \cdot \pi)}{4x - 8} = \left[\frac{\sin(3\pi)}{0} \right] = \frac{0}{0}$

$\lim\limits_{x \to \infty} \frac{\cos(5\pi - x\pi) \cdot (-\pi)}{4} = \frac{-1 \cdot (-\pi)}{4} = \pi/4$

c) $\lim\limits_{x \to \infty} \left(\sqrt[x]{e} \cdot \dfrac{3x^2 - 5x + 7}{x - 8 + 6x^2} \right)$

$\downarrow$

$1 \qquad \dfrac{x^2(3 - 5/x + 7/x^2)}{x^2(\frac{1}{x} - \frac{8}{x^2} + 6)} = 3/6 = 1/2$

21) $\lim\limits_{x \to -3} \dfrac{2x + 6}{2 \cdot \sqrt{4 - 4x} - (5 - x)} \xrightarrow{\text{Ableitung}} \dfrac{2}{2 \cdot \frac{1}{2\sqrt{4-4x}} \cdot (-4) + 1}$

Binom $\dfrac{2 \cdot (x+3) \cdot [2\sqrt{4-4x} + (5-x)]}{4 \cdot (4 - 4x) - (5-x)^2} = \dfrac{2 \cdot [2\sqrt{4-4x} + (5-x)]}{-(x+3)}$

$16 - 16x - 25 + 10x - x^2$

$-x^2 - 6x - 9$

$-(x^2 + 6x + 9)$

$-(x+3)^2$

$\Rightarrow \dfrac{32}{-0} = -\infty$

23)

$$f(x) = \begin{cases} x^2 + ax + b & ; x \geq 1 \\ x(2-b) + 2a & ; x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x + a & ; x \geq 1 \\ 2 - b & ; x < 1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1)$$

$$1 + a + b = (2-b) + 2a \qquad | -2 + b - a$$

$$\boxed{a = 2b - 1}$$

$$a = -\tfrac{1}{3}$$

$$\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x) = f'(1)$$

$$\boxed{2 + a = 2 - b} \qquad | -2 \qquad a = -b$$

$$2b - 1 = -b \qquad b = \tfrac{1}{3}$$

27) $f(x) = \frac{1}{2}x^3 - 3x^2 + \frac{9}{2}x + 42$

-> Extrempunkte

-> Wendetangente

$f'(x) = \frac{3}{2}x^2 - 6x + \frac{9}{2}$

$f''(x) = 3x - 6$

$f'''(x) = 3 <> 0 \Rightarrow$ WP existiert

$f'(x) = 0 = \frac{3}{2}x^2 - 6x + \frac{9}{2} \quad | \cdot \frac{2}{3}$

$x^2 - 4x + 3 = (x-1)(x-3) = 0$

$x_1 = 1 \qquad f(1) = \frac{1}{2} - 3 + \frac{9}{2} + 42 = 44 \rightarrow E_1(1/44)$

$x_2 = 3 \qquad f(3) = \frac{27}{2} - 27 + \frac{27}{2} + 42 = 42 \rightarrow E_2(3/42)$

$E_1(1/44)$ $\quad f''(1) = 3-6 = -3 < 0 \Rightarrow HP\,(1/44)$

$E_2(3/42)$ $\quad f''(3) = 9-6 = 3 > 0 \Rightarrow TP\,(3/42)$

$f''(x) = 0 = 3x - 6 \qquad x = 2$

$f(2) = \frac{8}{2} - 12 + 9 + 42 = 43 = y$

$f'(2) = \frac{3}{2} \cdot 4 - 12 + \frac{9}{2} = -\frac{3}{2} = m$

$y = m \cdot x + b \qquad 43 = -\frac{3}{2} \cdot 2 + b \qquad b = 45 + 1 = 46$

$t(x) = -\frac{3}{2} \cdot x + 46$

29) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$-16 \cdot \frac{2}{3} + 2 \cdot 6 + d = \frac{2}{3}$

$-\frac{32}{3} + \frac{36}{3} + d = \frac{2}{3}$

$d = -\frac{2}{3}$

$f'(3) = 0 \quad : \quad 27a + 6b + c = 0$

$f(2) = \frac{2}{3} \quad : \quad 8a + 4b + 2c + d = \frac{2}{3}$

$\underline{III} \quad f''(2) = 0 \quad : \quad 12a + 2b = 0 \longrightarrow b = -6a$

$f'(2) = -2 \quad : \quad 12a + 4b + c = -2 \qquad \underline{\underline{b = -4}}$

$$\left.\begin{array}{lll} \underline{I} & -9a + c = 0 \\ \underline{II} & -16a + 2c + d = \frac{2}{3} \\ \underline{IV} & -12a + c = -2 \end{array}\right\} \ominus \quad 3a + 0 = 2$$

$\underline{\underline{a = \frac{2}{3}}}$

$I \quad -8 + c = 2 \qquad \underline{\underline{c = 6}}$

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$$x^6 \to \frac{1}{7}x^7$$

d) $\displaystyle\int_\alpha^\infty \left(\frac{2}{(2x+3)^3}\right) dx = \frac{1}{2}$

$\hookrightarrow 2 \cdot (2x+3)^{-3} \Rightarrow \frac{2}{-2}\,\underbrace{(2x+3)^{-2}}_{1} \cdot \frac{1}{2}$

$\cdot(-\frac{1}{2})$

$G(x) = (2x+3)^{-2}$

$g(x) = -2\,(2x+3)^{-3} \cdot 2$

$= -4 \cdot \frac{1}{(2x+3)^3}$

$F(x) = -\frac{1}{2} \cdot (2x+3)^{-2}$

$F(x) = -\frac{1}{2} \cdot (2x+3)^{-2}$

$F(\infty) = -\frac{1}{2} \cdot \frac{1}{(2x+3)^2} = 0$

$F(\infty) - F(\alpha) = \frac{1}{2}$

$F(\alpha) = -\frac{1}{2} \cdot \frac{1}{(2\alpha+3)^2}$

$\frac{1}{2} \cdot \frac{1}{(2\alpha+3)^2} = \frac{1}{2} \quad | \cdot 2 \quad |^{-1} \qquad (2\alpha+3)^2 = 1 \quad | \sqrt{\ }$

$2\alpha + 3 = \pm 1 \qquad \alpha_1 = -2$

$\alpha_2 = -1$

31) $f(x) = 2 \cdot \sqrt{2x+5}$; $g(x) = x$

$2 \cdot (2x-5)^{1/2}$

$f(x) = g(x) \quad | -g(x) \quad \Rightarrow d(x) = 2 \cdot \sqrt{2x+5} - x$

$$2\sqrt{2x+5} = x \quad | \;\uparrow^2$$
$$4 \cdot (2x+5) = x^2$$
$$0 = x^2 - 8x - 20 = (x-10)(x+2) = 0$$
$$x_1 = 10 \quad \vee x_2 = -2$$

$$\int_{-2}^{10} d(x)\,dx = D(10) - D(-2)$$

$-\frac{1}{2}x^2$

$$D(x) = \frac{2}{3/2}(2x+5)^{3/2} \cdot \frac{1}{2} = \frac{2}{3} \cdot (2x+5)^{3/2} = \frac{2}{3}\sqrt{(2x+5)^3} - \frac{1}{2}x^2$$

$$D(10) = \frac{2}{3} \cdot \sqrt{(25)^3} - \frac{1}{2} \cdot 10^2 = \frac{2}{3} \cdot 125 - 50$$

$$D(-2) = \frac{2}{3} \cdot \sqrt{1^3} - \frac{1}{2} \cdot (-2)^2 = \frac{2}{3} - 2$$

$$\Rightarrow \frac{250}{3} - 50 - \frac{2}{3} + 2 = \frac{248}{3} - 48 = 82\tfrac{2}{3} - 48 = 34\tfrac{2}{3}$$