

$$2) \quad A = \{ \underline{1} ; \{2;3;4\} ; \underline{5} ; \{6;7\} ; \underline{8} \}$$

$$a) \quad X = \{ \underline{1} ; \underline{5} ; \underline{8} \} \quad \leftarrow \quad \cap = \{ \{5\} \}$$

$\{2;3;4;7\}$
 $\{5\}$

$$3) \quad \begin{aligned} & (X \vee (\overline{Y \wedge X})) \wedge ((Z \vee X) \vee (\bar{X} \vee Z)) \\ & (X \vee (\bar{Y} \vee \bar{X})) \wedge ((Z \vee X) \vee (\bar{X} \vee Z)) \\ & ((X \vee \bar{X}) \vee \bar{Y}) \wedge ((X \vee \bar{X}) \vee (Z \vee \bar{Z})) \\ & (1 \vee \bar{Y}) \wedge (1 \vee Z) \\ & 1 \wedge 1 \Rightarrow 1 \end{aligned}$$

$\downarrow$ de Morgan
 $\downarrow$ Ass. + Comm.
 $\downarrow$ Idempotenz
 $\downarrow$ Idempotenz

4) Potenzmengen = Menge aller Teilmengen

$$A = \{a, b, c\} \rightarrow 2^3$$

$$\mathcal{P}(A) = \left\{ \begin{array}{l} \{ \} \\ \{a\}, \{b\}, \{c\} \\ \{a, b\}, \{b, c\}, \{a, c\} \\ \{a, b, c\} \end{array} \right\}$$

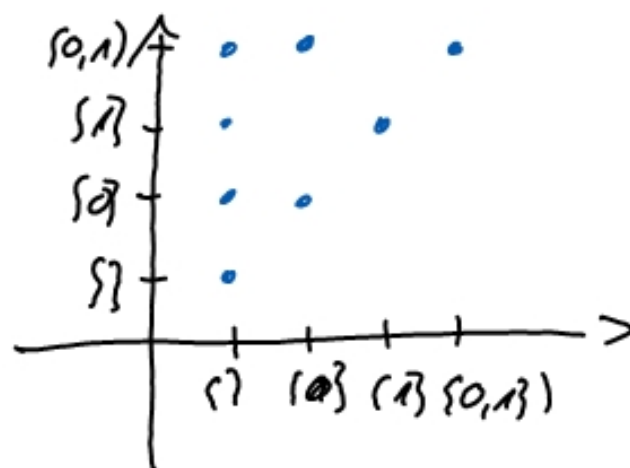
$$4) \quad \mathcal{P}(X) = \{ \emptyset, \{0\}, \{1\}, \{0,1\} \}$$

$$(\emptyset, \{0\}) \in \mathcal{P}(X) \times \mathcal{P}(X) \quad \checkmark$$

$$(\{1\}, \{0\}) \in \mathcal{P}(X) \times \mathcal{P}(X) \quad \nexists$$

$$\alpha \leq 6$$

$$\mathcal{R} = \{ (\emptyset, \emptyset), (\emptyset, \{1\}), (\emptyset, \{0\}), (\emptyset, \{0,1\}), \\ (\{0\}, \{0\}), (\{0\}, \{0,1\}), (\{1\}, \{0,1\}), (\{1\}, \{1\}), \\ (\{0,1\}, \{0,1\}) \}$$



$$6) A(p, q) = p \rightarrow q \stackrel{\downarrow}{\leftrightarrow} \neg p \vee q \quad \text{Bijection}$$

$$\left(\underbrace{(p \rightarrow q)}_{(a)} \wedge \underbrace{(\neg p \vee q)}_{(b)} \right) \vee \left(\neg(p \rightarrow q) \wedge \neg(\neg p \vee q) \right)$$

$$\underbrace{\left(\underbrace{(\neg p \vee q)}_{\substack{\neg \\ p}} \wedge \underbrace{(\neg p \vee q)}_{\substack{\neg \\ p}} \right)}_{\vee} \vee \left(\neg(\neg p \vee q) \wedge (p \wedge \neg q) \right)$$

$$\left((p \wedge \neg q) \wedge (p \wedge \neg q) \right)$$

$\vee$

$$7) \quad \heartsuit = \{ (a, b) \in \mathbb{N} \times \mathbb{N} \mid \sqrt{a} - \sqrt{b} = 2 \cdot k; k \in \mathbb{N} \}$$

$\downarrow$
 $\{0, 1, 2, \dots\}$

reflexive: $(a, a) \in \heartsuit; a \in \mathbb{N}$

$$\sqrt{a} - \sqrt{a} = 0 = 2 \cdot k \quad k = 0 \in \mathbb{N}$$

transitive: $(a, b) \in \heartsuit \wedge (b, c) \in \heartsuit \Rightarrow \underline{(a, c) \in \heartsuit}$

$$\begin{aligned}
 \sqrt{a} - \sqrt{b} &= 2 \cdot k_1 \quad \wedge \quad \sqrt{b} - \sqrt{c} = 2 \cdot k_2 \\
 \sqrt{a} - \sqrt{c} &= 2 \cdot k_1 + 2 \cdot k_2 \\
 \sqrt{a} - \sqrt{c} &= 2 \cdot (k_1 + k_2) = 2 \cdot k_3; k_3 \in \mathbb{N}
 \end{aligned}$$

antisymmetrisch $(4, 36) = \sqrt{4} - \sqrt{36} = 2 - 6 = -4 = 2 \cdot k$
 $k \in \mathbb{N}$

$$(36, 4) = \sqrt{36} - \sqrt{4} = 4$$

$$(a, s) \in \heartsuit \Rightarrow (s, a) \notin \heartsuit ; a \neq s$$

$$\underline{\sqrt{a} - \sqrt{s}} = 2 \cdot k_1 \quad \wedge \quad \sqrt{s} - \sqrt{a} = 2 \cdot k_2$$

$$\underline{\sqrt{a} - \sqrt{s}} = -2 \cdot k_2$$

$$2k_1 = -2k_2$$

$$k_1 = -k_2$$

$$\hookrightarrow k_1, k_2 \in \mathbb{N}$$

$$g) a) \quad f(x) = \sqrt{13-3x} \quad ; \quad g(x) = x-1 \quad ; \quad D = \mathbb{R}^{\leq 13/3}$$

$$\begin{aligned} \sqrt{13-3x} &= x-1 & | \uparrow^2 \\ 13-3x &= (x-1)^2 = x^2 - 2x + 1 & | -13 + 3x \end{aligned}$$

$$x^2 + x - 12 = 0 = (x+4)(x-3) \quad x_1 = -4 \quad \wedge \quad x_2 = 3$$

$$f(-4) = \sqrt{25} = 5 \quad g(x) = -4-1 = -5 \quad \swarrow$$

$$f(3) = \sqrt{4} = 2 \quad g(3) = 3-1 = 2 \quad \checkmark$$

$$L = \{3\} \quad \underline{\underline{S(3|2)}}$$

$$5) \frac{2x^2}{x^2+4x} - \frac{5x+4}{3x+12} = i(x) \quad ; \quad j(x) = \frac{x-5}{3x}$$

$$x \cdot (x+4) \quad 3 \cdot (x+4)$$

$$\mathbb{D} = \mathbb{R} \setminus \{-4; 0\}$$

$$\frac{2x^2}{x(x+4)} - \frac{5x+4}{3 \cdot (x+4)} = \frac{x-5}{3 \cdot x}$$

$$| \quad 3 \cdot x \quad (x+4)$$

$$6x^2 - x(5x+4) = (x-5)(x+4)$$

$$6x^2 - 5x^2 - 4x = x^2 - x - 20 \quad | -x^2$$

$$-3x = -20$$

$$x = \frac{20}{3} \quad \mathcal{L} = \left\{ \frac{20}{3} \right\}$$

$$10) \quad \mathcal{L} = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 2. y = e^{\alpha \cdot x} ; \alpha \in \mathbb{R}\}$$

$$y = A_2 \cdot e^{\alpha \cdot x}$$

$$y = \pm \sqrt{x}$$

Funktion: $f(x) = y_1 \wedge f(x) = y_2 \Rightarrow y_1 = y_2$

$$y_1 = A_2 \cdot e^{\alpha \cdot x} = A_2 e^{\alpha \cdot x} = y_2$$

$$y_1 = y_2$$

injektiv:

$$f(x_1) = y \wedge f(x_2) = y \Rightarrow x_1 = x_2$$

$$A_2 \cdot e^{\alpha x_1} = A_2 \cdot e^{\alpha x_2} \quad | : 2 \quad | \cdot$$

$$\alpha x_1 = \alpha x_2$$

$$x_1 = x_2$$

✓

Surjection $\{y \in \mathbb{R} \mid y = A e^{2x}; x \in \mathbb{R}\}$

$$y \in \mathbb{R}^+ \neq \mathbb{R} \neq \text{RAN}(\sim) \quad \text{⚡}$$

total : $\{x \in \mathbb{R} \mid y = A e^{2x}; y \in \mathbb{R}\}$

$$x \in \mathbb{R} \Leftrightarrow \mathbb{D} = \text{DOM}(\sim)$$

✓

$$11) \quad \boxed{9^n + 7} = 8 \cdot k \quad ; n \in \mathbb{N} ; k \in \mathbb{Z}$$

$$n=1 \quad 9^1 + 7 = 16 = 8 \cdot 2 \quad ; k=2 \in \mathbb{Z} \quad \checkmark$$

$$n+1 \quad 9^{n+1} + 7 = 8 \cdot k_1$$

$$9^1 \cdot 9^n + 7 = 8 \cdot k_1$$

$$\boxed{9^n + 7} + 8 \cdot 9^n = 8 \cdot k_1$$

$$8k_2 + 8 \cdot 9^n = 8 \cdot k_1$$

$$8(k_2 + 9^n) = 8 \cdot k_1$$

$$\mathbb{Z} + \mathbb{N} \rightarrow \mathbb{Z} \quad \checkmark$$

$$n) \quad 1 + n \cdot a \leq \boxed{(1+a)^n} ; \quad n \in \mathbb{N}$$

$$n=1 \quad 1 + 1 \cdot a \leq (1+a)^1$$

$$1+a \leq 1+a \quad \checkmark$$

$$n+1 \cdot 1 + (n+1) \cdot a \leq (1+a)^{n+1}$$

$$1 + n \cdot a + a \leq \boxed{(1+a)^n} \cdot (1+a)$$

$$\underline{1} + \underline{n \cdot a} + \underline{a} \leq (1+n \cdot a) \cdot (1+a) = \underline{1} + \underline{n \cdot a} + \underline{a} + n \cdot a^2$$

$$\left. \begin{array}{l} 0 \leq n \cdot a^2 \\ \downarrow \\ n \in \mathbb{N} \\ \geq 0 \end{array} \right\} \geq 0$$

$$13) \quad \underbrace{\sum (k-1) \cdot \ln \left(\frac{k}{k-1} \right)}_{a_n} = \underbrace{n \cdot \ln(n) - \ln(n!)}_{S_n}$$

$$n=2:$$

$$a_2 = S_2$$

$$(2-1) \cdot \ln \left(\frac{2}{2-1} \right) = 2 \cdot \ln(2) - \ln(2!)$$

$$\ln 2 = 2 \cdot \ln 2 - 1 \cdot \ln 2 \quad \checkmark$$

$$n: n+1$$

$$S_n + a_{n+1} = S_{n+1}$$

$$\underbrace{n \cdot \ln(n) - \ln(n!)}_{S_n} + \overbrace{\left((n+1)-1\right) \cdot \ln\left(\frac{n+1}{n}\right)}^{a_{n+1}} = \underbrace{(n+1) \cdot \ln(n+1) - \ln(n+1)!}_{S_{n+1}}$$

$$\begin{aligned}
 \underline{n \ln(n) - \ln(n!)} + n \cdot \ln\left(\frac{n+1}{n}\right) &= n \cdot \ln(n+1) + \underbrace{1 \cdot \ln(n+1)}_{\leftarrow} \\
 &= \ln\left[(n+1) \cdot n!\right] \leftarrow \\
 &= \left(\ln(n+1) + \ln(n!)\right) \leftarrow \\
 &= \ln(n+1) + \ln(n!) \leftarrow
 \end{aligned}$$

$$\begin{aligned}
 n \cdot \ln(n) + n \cdot \ln\left(\frac{n+1}{n}\right) &= n \cdot \ln(n+1) \\
 n \cdot \left[\ln\left[n \cdot \frac{(n+1)}{n}\right]\right] &= n \cdot \ln(n+1) \quad \left. \begin{array}{l} \nearrow \\ \searrow \end{array} \right\} = \checkmark
 \end{aligned}$$

$$14) \quad a_n = \sqrt{2} + e^{-2n}$$

$a_{n+1} - a_n \begin{cases} \nearrow > 0 \\ \searrow < 0 \end{cases}$

$$\sqrt{2} + e^{-2(n+1)} - [\sqrt{2} + e^{-2n}]$$

$$e^{-2n-2} - e^{-2n} = \frac{1}{e^{2n+2}} - \frac{1}{e^{2n}}$$

$$\frac{1}{e^2 \cdot e^{2n}} - \frac{1}{e^{2n}} = \frac{1 - e^2}{e^2 e^{2n}} = \frac{< 0}{> 0} = < 0$$

$$\frac{\sqrt{2} + e^{-2-2n}}{\sqrt{2} + e^{-2n}}$$

$\frac{a_{n+1}}{a_n} \begin{cases} \nearrow > 1 \\ \searrow \in [-1, 1] \end{cases}$

$$\frac{\sqrt{2} + \frac{1}{e^{2n+2}}}{\sqrt{2} + \frac{1}{e^{2n}}} = \frac{\frac{\sqrt{2} \cdot e^{2n+2} + 1}{e^{2n+2}}}{\frac{\sqrt{2} e^{2n} + 1}{e^{2n}}} = \frac{\sqrt{2} e^{2n+2} + 1}{e^{2n+2}} \cdot \frac{e^{2n}}{\sqrt{2} e^{2n} + 1}$$

$$\frac{\sqrt{2} e^{2n+2} + 1}{e^{2n+2}} \cdot \frac{e^{2n}}{\sqrt{2} e^{2n} + 1}$$

$$\frac{(e^2 \cdot e^{2n} \cdot \sqrt{2} + 1) \cdot e^{2n}}{e^2 \cdot e^{2n} \cdot (\sqrt{2} e^{2n} + 1)} = \frac{e^2 \cdot e^{2n} \sqrt{2} + 1}{e^2 \cdot e^{2n} \sqrt{2} + e^2} < 1$$

0, ...

$$\lim_{n \rightarrow \infty} (\sqrt{2} + e^{-2n}) = \lim_{n \rightarrow \infty} \left(\sqrt{2} + \frac{1}{e^{2n}} \right) = \sqrt{2}$$

$$\Rightarrow \text{Beschränktheit} \quad a_n \in [\sqrt{2} + 1; \sqrt{2}]$$

$$n \geq 8$$

15)

$$a_{n+1} = \sqrt{a_n} + 2 \quad ; \quad a_1 = 2,25 \quad ; \quad n \geq 1$$

$$a_1 = 2,25 \quad ; \quad a_2 = \sqrt{2,25} + 2 = 1,5 + 2 = 3,5$$

Behauptung: $a_{n+1} > a_n$

$n=1$

$$a_2 > a_1$$

$$3,5 > 2,25 \quad \checkmark$$

$n+1$

$$a_{n+2} > a_{n+1}$$

$$\sqrt{a_{n+1}} + 2 > \sqrt{a_n} + 2 \quad | -2$$

$$\sqrt{a_{n+1}} > \sqrt{a_n} \quad | \uparrow ?$$

$$a_{n+1} > a_n$$

Beschränktheit: $a_1 = 2,25$ ist untere Schranke

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1}$$

Behauptung: $a_n < 4$

$$\begin{aligned} n=1 & \quad a_1 = 2,25 < 4 \quad \checkmark \\ n+1: & \quad a_{n+1} < 4 \quad (\text{zu zeigen}) \\ & \quad \sqrt{a_n} < \sqrt{4} = 2 \end{aligned}$$

$$\begin{aligned} \sqrt{a_n} + 2 & < 4 \\ \underbrace{\phantom{\sqrt{a_n} + 2}}_{a_{n+1}} & < 4 \end{aligned}$$

$$\begin{aligned} x &= \sqrt{x} + 2 & | -2 \\ (x-2)^2 &= \sqrt{x} & | ()^2 \\ x^2 - 4x + 4 &= x & | -x \end{aligned}$$

$$\begin{aligned} x^2 - 5x + 4 &= 0 \\ (x-4)(x-1) &= 0 \\ x_1 = 4 & \cup x_2 = 1 \end{aligned}$$

$$1+2$$

$$\sum_{k=2}^{\infty} (-1)^{k+1} \cdot \frac{3 \cdot 9^k}{(2k+1)!}$$

$$\sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!}$$

$\underbrace{\hspace{10em}}_{\sin(x)}$

$$\sum_{k=2}^{\infty} (-1)^k \cdot (-1)^1 \cdot \frac{3^{2k+1}}{(2k+1)!}$$

$$\begin{aligned} 3 \cdot 9^k &= 3 \cdot (3^2)^k \\ &= 3^1 \cdot 3^{2k} = 3^{2k+1} \end{aligned}$$

$$- \left[\sin(3) - \left(\underbrace{(-1)^0 \cdot \frac{3^{0+1}}{(0+1)!}}_{a_0} + \underbrace{(-1)^1 \cdot \frac{3^{2+1}}{(2+1)!}}_{a_1} \right) \right]$$

$$- \left[\sin(3) - \left(3 - \frac{9}{2} \right) \right] = -\sin(3) - 1,5$$

$$\sum \frac{k^4 \cdot x^k}{\sqrt{42k!}}; k \in \mathbb{N}$$

$$\lim_{k \rightarrow \infty} \frac{\frac{(k+1)^4 \cdot x^{k+1}}{\sqrt{42 \cdot (k+1)!}}}{\frac{k^4 \cdot x^k}{\sqrt{42k!}}}$$

$$\left(\frac{k+1}{k}\right)^4 \cdot \frac{x^{k+1}}{x^k} \cdot \sqrt{\frac{42k!}{42 \cdot (k+1)!}}$$

$$\underbrace{\left(1 + \frac{1}{k}\right)^4}_1 \cdot x \cdot \underbrace{\sqrt{\frac{1}{(k+1)}}}_0$$

$$1 \cdot x \cdot 0 = 0 < 1$$

$x \in \mathbb{R}$ (Kongruent) ✓

$$19) a) \lim_{x \rightarrow \infty} \left(\frac{3}{x} - \left(1 + \frac{3}{x} \right)^x \right) \quad \left(1 + \frac{x}{x} \right)^x \rightarrow e^x$$

$$\downarrow$$

$$0 - e^3 \rightarrow -e^3$$

$$b) \lim_{x \rightarrow 2} \frac{\cos(5\pi - x\pi)}{4x - 8} = \frac{-1}{0} = -\infty$$

$$c) \lim_{x \rightarrow \infty} \left(x \sqrt[3]{e} \cdot \frac{3x^3 - 5x + 7}{x - 8 + 6x^2} \right)$$

$$\downarrow$$

$$1 \quad \rightarrow \frac{x^3 (3 - 5/x + 7/x^2)}{x^2 (1/x - 8/x^2 + 6)} = \frac{3}{6} \quad \left. \vphantom{\frac{x^3 (3 - 5/x + 7/x^2)}{x^2 (1/x - 8/x^2 + 6)}} \right\} 1/2$$

$$21) \quad \lim_{x \rightarrow 0} \left(\frac{2x}{\frac{1}{3}\sqrt{9+2x} - 1} \right) = \frac{0}{0}$$

$$a) \quad \lim_{x \rightarrow 0} \frac{2}{\frac{1}{3} \frac{1}{2 \cdot \sqrt{9+2x}} \cdot 2} = \lim_{x \rightarrow 0} \frac{2}{\frac{1}{3 \cdot \sqrt{9+2x}}} = \underline{\underline{18}}$$

$$b) \quad \frac{2x}{\frac{1}{3}\sqrt{9+2x} - 1} \cdot \frac{\frac{1}{3}\sqrt{9+2x} + 1}{\frac{1}{3}\sqrt{9+2x} + 1} = \frac{2x \cdot (\frac{1}{3}\sqrt{9+2x} + 1)}{\frac{1}{9} \cdot (9+2x) - 1}$$

$$\lim_{x \rightarrow 0} \frac{2 \cdot (\frac{1}{3}\sqrt{9+2x} + 1) \cdot \frac{9}{2}}{\frac{1}{9} \cdot (9+2x) - 1} = \underline{\underline{18}}$$

23)

$$f(x) = \begin{cases} x^2 + ax + b & x \geq 1 \\ x(2-b) + 2a & x < 1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} f(x) = f(1) = \lim_{x \rightarrow 1^-} f(x)$$

$$1 + a + b = 1 + a + b = 2 - b + 2a$$

$$-a + 2b = 1$$

$$f'(x) = \begin{cases} 2x + a & ; x \geq 1 \\ 2 - b & ; x < 1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} f'(x) = f'(1) = \lim_{x \rightarrow 1^-} f'(x)$$

$$2 + a = 2 + a = 2 - b \Rightarrow a = -b$$

$$-(-b) + 2b = 3b = 1$$

$$\underline{\underline{b = \frac{1}{3}}}$$

$$\underline{\underline{a = -\frac{1}{3}}}$$

$$27) \quad f(x) = \frac{1}{2}x^3 - 3x^2 + \frac{9}{2}x + 42$$

$$f'(x) = \frac{3}{2}x^2 - 6x + \frac{9}{2}$$

$$f''(x) = 3x - 6 = 0 \Rightarrow x = 2$$

$$f'(2) = 6 - 12 + \frac{9}{2} = -\frac{3}{2} = m$$

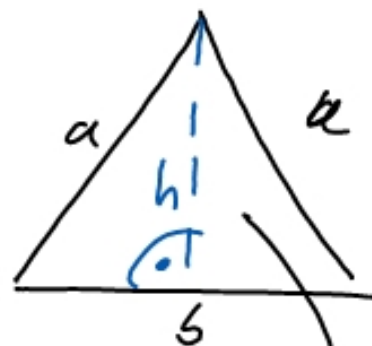
$$f(2) = 4 - 12 + 9 + 42 = 43 = \underline{y}$$

$$y = m \cdot x + b$$

$$43 = -\frac{3}{2} \cdot 2 + b \Rightarrow b = 46$$

$$y = -\frac{3}{2}x + 46$$

$$28) \quad u = 6 \text{ cm}$$



$$A = \frac{1}{2} \cdot g \cdot h$$

$$6 = 2a + s$$

$$A = \frac{1}{2} \cdot s \cdot h$$

$$A = \frac{1}{2} \cdot s \cdot \sqrt{a^2 - \frac{s^2}{4}}$$

$$\rightarrow (3 - \frac{1}{2}s) = a$$

$$A = \sqrt{\frac{1}{4}s^2 \cdot \left[\left(3 - \frac{s}{2}\right)^2 - \frac{s^2}{4} \right]}$$

$$A(s) = \sqrt{\frac{1}{4}s^2 \left(9 - 3s + \frac{s^2}{4} - \frac{s^2}{4} \right)}$$

$$h^2 + \left(\frac{s}{2}\right)^2 = a^2$$

$$h^2 = a^2 - \frac{s^2}{4}$$

$$h = \sqrt{a^2 - \frac{s^2}{4}}$$

$$A(s) = \frac{1}{2} \cdot \sqrt{9s^2 - 3s^3}$$

$$A'(s) = \frac{1}{2} \cdot \frac{1}{2 \cdot \sqrt{9s^2 - 3s^3}} \cdot (18s - 9s^2)$$

$$= \frac{18s - 9s^2}{4 \cdot \sqrt{9s^2 - 3s^3}} = 0 \quad \begin{aligned} 18s - 9s^2 &= 0 \\ 9s(2 - s) &= 0 \\ s &= 0 \vee s = 2 \end{aligned}$$

$$f'(2) = 0$$

$$s=2 \quad a=2$$

$$f'(1) = \frac{18-9}{>0} > 0$$

$$f'(3) = \frac{54-81}{>0} < 0$$

Ans

$$A(2) = \frac{1}{2} \cdot \sqrt{36 - 24} = \frac{1}{2} \cdot \sqrt{12} = \sqrt{3}$$

29) $x=3$: Treppunkt
 $(2, \frac{7}{3})$: Wendepunkt
 $\hookrightarrow$ Steigung -2

$$\begin{aligned}f(x) &= ax^3 + bx^2 + cx + d \\f'(x) &= 3ax^2 + 2bx + c \\f''(x) &= 6ax + 2b\end{aligned}$$

$$f'(3) = 0$$

$$0 = 27a + 6b + c$$

$$f(2) = \frac{7}{3}$$

$$\frac{7}{3} = 8a + 4b + 2c + d$$

$$f'(2) = 0$$

$$0 = 12a + 2b \quad \Rightarrow \quad b = -6a$$

$$f'(2) = -2$$

$$-2 = 12a + 4b + c$$

$$2 = 15a + 2b$$

$$2 = 15a - 12a$$

$$2 = 3a \quad \Rightarrow \quad a = \frac{2}{3}$$

$$b = -4$$

$$a = \frac{2}{3} \quad b = -4$$

$$-2 = 12c + 4b + c$$

$$r_3 = 8a + 4b + 2c + d$$

↑

ACHT

$$-2 = 8 - 16 + c$$

$$c = 6$$

$$r_3 = \frac{16}{3} - 16 + 12 + d$$

$$d = -\frac{2}{3}$$

$$f(x) = \frac{2}{3}x^3 - 4x^2 + 6x - \frac{2}{3}$$

$$30) 5) \quad \int_1^3 \underbrace{(x^2 - 6x + 8)}_{(x-4)(x-2)} dx = \int_1^2 f(x) dx + \int_2^3 f(x) dx$$

$$F(x) = \frac{1}{3}x^3 - 3x^2 + 8x$$

$$F(3) = 9 - 27 + 24 = 6$$

$$F(2) = \frac{8}{3} - 12 + 16 = \frac{20}{3}$$

$$F(1) = \frac{1}{3} - 3 + 8 = \frac{16}{3}$$

$$|F(3) - F(2)| + |F(2) - F(1)|$$

$$|\frac{18}{3} - \frac{20}{3}| + |\frac{20}{3} - \frac{16}{3}| = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = \underline{\underline{2}}$$

$$31) \quad f(x) = 2 \cdot \sqrt{2x+5} \quad ; \quad g(x) = x$$

1. Schnittstellen

$$f(x) = g(x)$$

$$2\sqrt{2x+5} = x \quad | \uparrow^2$$

$$4 \cdot (2x+5) = x^2$$

$$x^2 - 8x - 20 = 0$$

$$x_1 = 10$$

$$(x-10)(x+2) = 0 \quad x_2 = -2$$

2. Differenzfunktion:

$$2\sqrt{2x+5} - x = d(x)$$

$$d(x) = 2 \cdot (2x+5)^{1/2}$$

$$= \frac{2}{3} (2x+5)^{3/2}$$

$$d'(x) = \frac{3}{2} \cdot \frac{2}{3} \cdot (2x+5)^{1/2} \cdot 2 = 2 \cdot \sqrt{2x+5}$$

$$\int_{-2}^{10} d(x) dx = |D(10) - D(-2)| = 8$$

~~$$D(10) = 2 \cdot \sqrt{2 \cdot 10 + 5} = 2 \cdot \sqrt{25} = 10$$~~

~~$$D(-2) = 2 \cdot \sqrt{1} = 2 \cdot 1 = 2$$~~

$$D(x) = \frac{2}{3} \cdot \sqrt{(2x+5)^3} \Rightarrow \frac{2}{3} \cdot (2x+5)^{3/2} \cdot 2$$

$$\left. \begin{aligned} D(10) &= \frac{2}{3} \cdot \sqrt{(25)^3} = \frac{250}{3} \\ D(-2) &= \frac{2}{3} \cdot \sqrt{1^3} = \frac{2}{3} \end{aligned} \right\} \Delta = \frac{248}{3}$$