$$a_n = \left(\tfrac{1}{3}\right)^n - 3 \quad ; \quad n \geq 0$$

$$a_0 = 1 - 3 = -2 \quad ; \quad a_1 = \tfrac{1}{3} - 3 = -2\tfrac{2}{3} \qquad \downarrow$$

$$a_{n+1} < a_n$$

$$n = 0 \qquad a_1 < a_0 \quad : \quad -2\tfrac{2}{3} < -2 \qquad \checkmark$$

$$n+1 \qquad a_{n+2} < a_{n+1}$$

$$\left(\tfrac{1}{3}\right)^{n+2} - 3 < \left(\tfrac{1}{3}\right)^{n+1} - 3 \qquad | +3$$

$$\left(\tfrac{1}{3}\right)^{n} \cdot \left(\tfrac{1}{3}\right)^{2} < \left(\tfrac{1}{3}\right)^{n} \cdot \left(\tfrac{1}{3}\right)^{1} \qquad | : \left(\tfrac{1}{3}\right)^{n}$$

$$\tfrac{1}{9} < \tfrac{1}{3} \qquad \checkmark$$

$a_0 = -2$ ist obere Schranke

$a_n > -3$ (untere Schranke)

$n=0$ $\quad a_0 = -2 > -3$ ✓

$n+1$ $\quad a_{n+1} > -3$

$$\left(\tfrac{1}{3}\right)^{n+1} - 3 > -3 \qquad | +3$$

$$\left(\tfrac{1}{3}\right)^{n} \cdot \left(\tfrac{1}{3}\right)^{1} > 0 \qquad | \cdot 3$$

$$\left(\tfrac{1}{3}\right)^{n} > 0 \qquad | \sqrt[n]{\ }$$

$$\tfrac{1}{3} > 0 \qquad ✓$$

$$\lim_{n \to \infty} \left(\tfrac{1}{3}\right)^{n} - 3 = [0 - 3] = \underline{\underline{-3}}$$

1) $\sum_{k=3}^{10} 5\cdot(2{,}5)^{-k}$

$\boxed{2)\ \sum_{k=3}^{\infty} \frac{2^{2k+1}}{3\cdot k!}}$

$5\cdot\sum_{k=3}^{10}\left(2/5\right)^{k}$

$5\cdot\left[-\sum_{k=0}^{2}\left(2/5\right)^{k}+\sum_{k=0}^{10}\left(2/5\right)^{k}\right] = 5\cdot\left(-\frac{1-(2/5)^{3}}{1-2/5}+\frac{1-(2/5)^{11}}{1-2/5}\right)$

$5\cdot\frac{5}{3}\cdot\left[-(1-(2/5)^{3}+1-(2/5)^{11}\right] = \frac{25}{3}\cdot\left((2/5)^{3}-(2/5)^{11}\right)$

$5\cdot\left[\sum_{k=0}^{10}\left(2/5\right)^{k}-\left(1+2/5+4/25\right)\right]$

$5\cdot\left(\frac{1-(2/5)^{11}}{1-2/5}-\frac{39}{25}\right) = \frac{25}{3}\cdot\left(1-(2/5)^{11}\right)-\frac{39}{5}$

$$\sum_{k=3}^{\infty} \frac{2^{2k} \cdot 2^1}{3 \cdot k!} = \frac{2}{3} \cdot \sum_{k=3}^{\infty} \frac{4^k}{k!}$$

$$= \frac{2}{3} \left[ e^4 - \left( \frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} \right) \right]$$

$$= 2/3 \cdot \left[ e^4 - (1 + 4 + 8) \right]$$

$$= 2/3\, e^4 - 26/3$$

$$\lim_{x \to 3} \frac{3x - 9}{2 - \sqrt{2x - 2}}$$

2 Arten

Binom

L'Hospital

a) $$\lim_{x \to 3} \frac{3x-9}{\underset{a}{2} - \underbrace{\sqrt{2x-2}}_{b}} \cdot \frac{2+\sqrt{2x-2}}{2+\sqrt{2x-2}}$$

$$\lim_{x \to 3} \frac{(3x-9)\cdot(2+\sqrt{2x-2})}{4-(2x-2)} = \frac{\overbrace{(3x-9)}^{0}(2+\sqrt{2x-2})}{\underbrace{-2x+6}_{0}}$$

$$\frac{3(x\cancel{-3})\cdot(2+\sqrt{2x-2})}{-2(x\cancel{-3})} = -\frac{3}{2}\cdot\left(2+\sqrt{2x-2}\right) = \underline{\underline{-6}}$$

b) $$\lim_{x \to 3} \frac{3}{-\frac{1\cdot 2}{2\cdot\sqrt{2x-2}}} = 3\cdot\left(-\sqrt{2x-2}\right) = -6$$

$$(2x-2)^{1/2} \to 1/2\,(2x-2)^{-1/2}\cdot 2 = \frac{1}{\sqrt{2x-2}}$$

1) $f(x) = \begin{cases} x^2 + a & ; x \geq 1 \\ x(4-b) & ; x < 1 \end{cases}$ stetig + differenzierbar ?

2) $f(x) = 2 \cdot \cos\left(\frac{1}{2}x + 7{,}5\pi\right) - 4$

Amplituden (1W) ; Symmetrie ; Periode

stetig : $1^2 + a = 1(4-b) \iff 1 + a = 4 - b \quad a = 1$

diff : $2 \cdot 1 = 4 - b \iff b = 2$

$$f(x) = 2\cdot\left[\cos \tfrac{1}{2}x \cdot \underbrace{\cos 7{,}5\pi}_{0} - \sin \tfrac{1}{2}x \cdot \underbrace{\sin 7{,}5\pi}_{-1}\right] - 4$$

$$f(x) = 2\cdot \sin(\tfrac{1}{2}x) - 4$$

$$P_{NEU} = \frac{P_{ALT}}{b} = \frac{2\pi}{1/2} = 4\pi \quad \Rightarrow \quad f(x) = f(x+4\pi)$$

$$2\sin(\tfrac{1}{2}x) - 4 = 2\cdot\sin(\tfrac{1}{2}(x+4\pi)) - 4 \quad |+4 \cdot \tfrac{1}{2}$$

$$\sin(\tfrac{1}{2}x) = \sin(\tfrac{1}{2}x + 2\pi)$$

$$= \sin(\tfrac{1}{2}x)\cdot\underbrace{\cos(2\pi)}_{1} + \cos(\tfrac{1}{2}x)\cdot\underbrace{\sin 2\pi}_{0}$$

$$= \sin(\tfrac{1}{2}x) \quad \checkmark$$

GERADE

$\begin{pmatrix}\sin \\ \cos\end{pmatrix}$ $\rightarrow P_{ALT} = \pi$

$\rightarrow [0;1]$

$$W: 2\cdot[-1;1] - 4 = [-2;2] - 4 = [-6;-2]$$

$$W = y \in [-6;-2]$$

<u>Symmetrie</u> : $\sin$ : $f(x) = -f(-x)$

Punktsymmetrie: $(0|-4)$

$$f(x) = \frac{8}{\sqrt{(0{,}5x+2)^{-3}}} \; ; \; P_3(x,4) \qquad f(x)+4 = -[f(-x)+4]$$

$$\sqrt[n]{x} = x^{1/n}$$

$$(2\cdot\sin(\tfrac{1}{2}x)-4)+4 = -[(2\cdot\sin(-\tfrac{1}{2}x)+4)-4]$$

$$2\cdot\sin(\tfrac{1}{2}x) = -2\cdot\sin(-\tfrac{1}{2}x) \quad |\cdot\tfrac{1}{2}$$

$\alpha = \frac{1}{2}x$

$$\sin(\tfrac{1}{2}x) = -\sin(-\tfrac{1}{2}x)$$

$$\sin(\alpha) = -\sin(-\alpha) \quad ✓$$

$$f(x) = \frac{8}{\left((0{,}5x+2)^{1/2}\right)^{-3}}$$

$$= \frac{8}{(0{,}5x+2)^{-3/2}} = 8\cdot(0{,}5x+2)^{3/2}$$

| $n$ | $f^{n}(x)$ | $f^{n}(4)$ | $(x-4)^{n}$ | $n!$ |
|---|---|---|---|---|
| 0 | $8 \cdot (0{,}5x+2)^{3/2}$ | $8 \cdot 2^{3} \Rightarrow 64$ | $1$ | $1$ |
| 1 | $6 \cdot (0{,}5x+2)^{1/2}$ | $6 \cdot 2^{1} \rightarrow 12$ | $(x-4)$ | $1$ |
| 2 | $3/2\, (0{,}5x+2)^{-1/2}$ | $3/2 \cdot 2^{-1} \rightarrow 3/4$ | $(x-4)^{2}$ | $2$ |
| 3 | $-3/8\, (0{,}5x+2)^{-3/2}$ | $-3/8 \cdot 2^{-3} \rightarrow -3/64$ | $(x-4)^{3}$ | $6$ |

$$P_3(x,4) = 64 + 12\,(x-4) + 3/8\,(x-4)^2 - 1/128\,(x-4)^3 + R_3(x,4)$$

$$\mathbb{D} = \mathbb{R}^{\geq -4}, \quad \mathbb{W} = \mathbb{R}_0^+$$

1) $$\underbrace{\int (x^3 - 8x^2 - 20x)\, dx}_{A \text{ zw. Funktion + x-Achse}}$$

2) $$\int_0^2 x^2 \cdot e^{2x}\, dx$$

$$\int (x^3 - 8x^2 - 20x)\,dx = \int x(x-10)(x+2)\,dx$$

$$\Rightarrow |F(0) - F(-2)| + |F(10) - F(0)|$$

$$F(x) = \tfrac{1}{4}x^4 - \tfrac{8}{3}x^3 - 10x^2$$

$$F(0) = 0$$

$$F(-2) = 4 + \tfrac{64}{3} - 40 \quad \Big\} \quad \frac{44}{3} = 14\tfrac{2}{3} = 14{,}\overline{66}$$

$$F(10) = \underbrace{\tfrac{1}{4}\cdot 10^4}_{2500} - \tfrac{8}{3}\cdot 10^3 - 10^3 \quad \Big\} \quad \underline{\underline{1180}}$$

$$2500 - \tfrac{11}{3}\cdot 10^3 = 2500 - 3666 = 1166$$

$$\int_0^2 \underbrace{x^2}_{g}\, \underbrace{e^{2x}}_{f'}\, dx$$

$$\boxed{\int f' \cdot g = f \cdot g - \int f \cdot g'}$$

$f'(x) = e^{2x} \rightarrow f(x) = \frac{1}{2} e^{2x}$

$g(x) = x^2 \rightarrow g'(x) = 2x$

$\frac{1}{2} x^2 e^{2x} - \int \frac{1}{2} e^{2x} \cdot 2x \, dx$

$f'(x) = e^{2x} \rightarrow f(x) = \frac{1}{2} e^{2x}$

$g(x) = x \rightarrow g'(x) = 1$

$\frac{1}{2} x^2 e^{2x} - \left[ \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \, dx \right]$

$\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} \Big|_0^2$

$F(2) - F(0)$

$\frac{1}{4} \cdot (5e^4 - 1)$

$F(2) = 2e^4 - e^4 + \frac{1}{4} e^4$

$= \frac{5}{4} e^4$

$F(0) = \frac{1}{4}$