1) $A = \{ x \in \,]4;14[_{\mathbb{N}} \mid x \bmod 3 = 0 \}$

$[5;13]_{\mathbb{N}}$

$A = \{6; 9; 12\}$

a) $A \cap B = \{9; 12\}$ $\Leftarrow$

$= \{ x \in [9;12]_{\mathbb{N}} \mid x \bmod 3 = 0 \}$

b) $A \cup B = \{ x \in [6;20]_{\mathbb{N}} \mid x \neq 8 \vee x \neq 10 \}$

$= \{ x \in \mathbb{N} \setminus \{8, 10\} \mid x \geq 6 \wedge x \leq 20 \}$

c) $A \setminus B = \{6\} = \{ x \in \mathbb{N} \mid x > 5 \wedge x < 7 \}$

d) $B \setminus A = \{7; 11; 13; 14; 15 \ldots ; 18; 19; 20\}$

$\{ x \in \mathbb{N} \setminus \{8; 9; 10; 12\} \mid x \geq 7 \wedge x \leq 20 \}$

4) a)

$$\ln\left[\frac{x^{2}\,y^{1/2}}{4\,x^{1/2}\,y^{-2}}\right]^{1/3} = \ln\left[\frac{x^{3/2}\,y^{5/2}}{4}\right]^{1/3}$$

$$\ln\left(\frac{x^{1/2}\,y^{5/6}}{4^{1/3}}\right) = \tfrac{1}{2}\ln x + \tfrac{5}{6}\ln y - \tfrac{1}{3}\ln 4$$

5)

$$\frac{2^{8}\,a^{8}\,b^{-12}\,c^{24}\quad 2^{-3}\,a^{-3}\,b^{-12}\,c^{-6}}{2\cdot 2^{6}\,d^{4}\,c^{-4}\,e^{6}\quad 2^{-4}\,a^{8}\,d^{-4}\,e^{-8}}$$

$$\frac{2^{4}\,2^{8}\,a^{8}\,c^{4}\,c^{4}\,d^{4}\,e^{8}}{2^{3}\,2^{7}\,b^{12}\,a^{3}\,b^{12}\,c^{6}\,d^{4}\,e^{6}\,a^{8}}$$

$$\frac{4\,c^{2}\,e^{2}}{a^{3}\,b^{24}}$$

c) $$\left[\frac{a^{\frac{5-n}{n}}}{a^{\frac{2n+4}{2n}}} \cdot a^{\frac{3n+3}{n}} \right]^5 \qquad \frac{2n+4}{2n} \rightarrow \frac{\cancel{2}(n+2)}{\cancel{2}\cdot n}$$

$$\left(a^{\frac{5-n-n-2+3n+3}{n}} \right)^5 = \left(a^{\frac{6+n}{n}} \right)^5$$

$$a^{\frac{30+5n}{n}}$$

d) $$0{,}5 \cdot \mathrm{ld}\,(2^2)^3 + 2 \cdot e^{\ln 2^3} - 4 \cdot \left[\log 10^{3/2} - 2^{\mathrm{ld}\,(4^{1/3})^3} \right]$$

$$+ \ln e^{-3} - 2 \cdot 10^{\log 16}$$

$$= \underbrace{3 + 16}_{19} \underbrace{- 4 \cdot \left(3/2 - 4 \right)}_{+10} \underbrace{- 3 - 32}_{-35} = -6$$

5) a) $(2-\boxed{i})^3 \cdot (2i^2)^3$

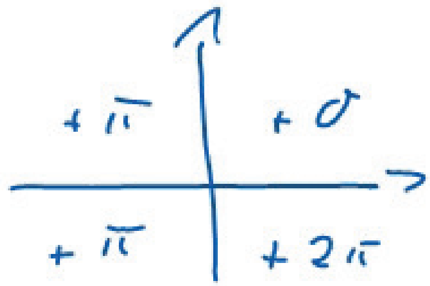
$(2-11i)\cdot(-2)^3$

$$\begin{array}{ccccccc|c}
 & & & 1 & & & & 0 \\
 & & 1 & & 1 & & & 1 \\
 & 1 & & 2 & & 1 & & 2 \\
1 & & 3 & & 3 & & 1 \leftarrow & 3
\end{array}$$

$$\overset{+}{1}\,2^3\,\boxed{i^0} \overset{-}{-} 3\,2^2\,\boxed{i^1} \overset{+}{+} 3\,2^1\,\boxed{i^2} \overset{-}{-} 1\,2^0\,\boxed{i^3}$$

$$8 \quad -12i \quad -6 \quad +i \Rightarrow -11i + 2$$

$$\Rightarrow -8\cdot(2-11i) = 88i - 16$$



$$r = \sqrt{88^2 + 16^2}$$

$$\alpha = \arctan\left(-\frac{88}{16}\right) + \pi$$

5) $$z = \frac{3i-2}{2+i}\cdot\frac{2-i}{2-i} + \frac{3i}{2+3i}\cdot\frac{2-3i}{2-3i}$$

$$= \frac{6i-4+2i-3i^2}{5} + \frac{6i+9}{13}$$

$$= \frac{13\cdot(8i-1)+5\cdot(6i+9)}{65}$$

$$= \frac{104i-13+30i+45}{65} = \frac{134i+32}{65} = \frac{134}{65}i + \frac{32}{65}$$

6) $$z\cdot\left(z^2 + \underbrace{(1-2i)}_{p}\cdot z - \underbrace{(i+1)}_{q}\right) = 0 \qquad z_1 = 0$$

$$z_{2/3} = -\frac{1-2i}{2} \pm \sqrt{\left(\frac{1-2i}{2}\right)^2 + i + 1}$$

$$z_{2/3} = -\frac{1-2i}{2} \pm \sqrt{\frac{1-4i-4}{4} + \frac{4i+4}{4}}$$

$$z_{2/3} = -\frac{1-2i}{2} \pm \sqrt{\frac{1}{4}}$$

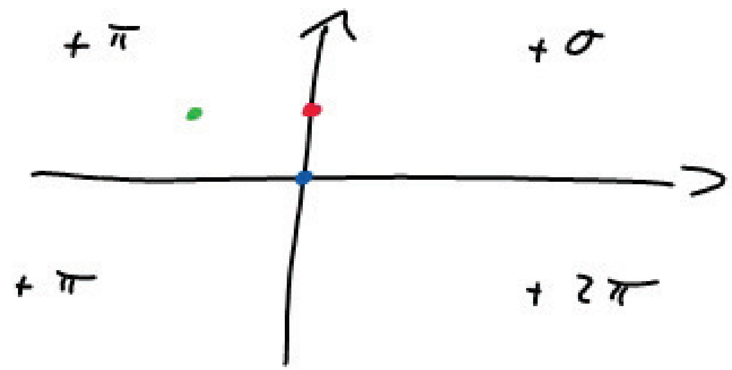
$$z_2 = -\frac{1}{2} + i + \frac{1}{2} = i$$

$$z_3 = -\frac{1}{2} + i - \frac{1}{2} = i - 1$$

$\underline{z_1} = 0 \qquad r = 0 \qquad \alpha = 0$

$\underline{\underline{z_2}} = i \qquad r = 1 \qquad \alpha = \pi/2$

$\underline{\underline{z_3}} = i - 1 \qquad r = \sqrt{2} \qquad \alpha = \arctan(-1) + \pi$



Definitionsbereich:

$\to \ln(\heartsuit) \Rightarrow \heartsuit > 0$

$\to \sqrt{\heartsuit} \Rightarrow \heartsuit \geq 0$

$\to \frac{\sim}{\heartsuit} \Rightarrow \heartsuit \neq 0$

7) a) $\sqrt{13-3x} = x-1 \quad |^2$ $\qquad 13-3x \geq 0 \Leftrightarrow -3x \geq -13 \Rightarrow x \leq 13/3$

$13-3x = (x-1)^2$ $\qquad \mathbb{D} = \{x \in \mathbb{R} \mid x \leq 13/3\}$

$0 = (x^2-2x+1) - 13 + 3x$

$= x^2 + x - 12 = (x+4)(x-3) = 0 \qquad x_1 = 3 \lor x_2 = -4$

$\mathbb{L} = \{-4; 3\}$

b) $x^3 - 7x + 6 = 0 \quad ; \quad \mathbb{D} = \mathbb{R}$

$(x^3 - 7x + 6)(x-1) = \underbrace{x^2 + x - 6}_{(x+3)\cdot(x-2)}$

$-(x^3 - x^2)$

$x^2 - 7x + 6$

$-(x^2 - x)$

$-6x + 6$

$\mathbb{L} = \{-3; 1; 2\}$

c) $z \cdot (z^4 - 50z^2 + 49) = z \cdot (z^2 - 49) \cdot (z^2 - 1)$

$\Rightarrow \mathbb{D} = \mathbb{R} \quad \wedge \quad \mathbb{L} = \{0, \pm 1; \pm 7\}$

$x^2 = z \qquad x^2 - 50x + 49 = 0$

$(x-49)(x-1) = 0$

$x_{1/2} = 25 \pm \sqrt{25^2 - 49}$

$25 \pm \sqrt{576} = 25 \pm 24$

$x_1 = 49 \qquad x_2 = 1$

$z_{2/3} = \pm\sqrt{49} = \pm 7$

$z_{4/5} = \pm\sqrt{1} = \pm 1$

d) $$\frac{2x^2}{x\cdot(x+4)} - \frac{5x+4}{3(x+4)} = \frac{x-5}{3x} \quad \Big| \cdot 3 \cdot x \cdot (x+4)$$

$\cancel{x} \cdot \cancel{(x+4)}$

$\cancel{3} \cdot \cancel{(x+4)}$

$\cancel{(3)}\ \cancel{(x)}$

$$2\cdot x^2\cdot 3 - (5x+4)\cdot x = (x-5)\cdot(x+4)$$

$$6x^2 - 5x^2 - 4x = x^2 - x - 20$$

$$-3x = -20$$

$$x = 20/3$$

$$\mathbb{D} = \mathbb{R} \setminus \{0; -4\} \Rightarrow \mathbb{L} = \{20/3\}$$

8) a) $x^2 - 8x - 48 = 0$ pr Q.E.

$$(x-4)^2 - 4^2 - 48 = 0$$
$$(x-4)^2 - 64 = 0$$
$$(x-4)^2 = 64$$
$$x-4 = \pm\sqrt{64} = \pm 8 \quad \rightarrow x_1 = 12, \quad \rightarrow x_2 = -4$$

b) $x^2 + \frac{1}{6}x - \frac{1}{6} = 0$ p-q-Formel

$$x_{1/2} = -\frac{1}{12} \pm \sqrt{\frac{1}{144} + \frac{24}{144}} = -\frac{1}{12} \pm \sqrt{\frac{25}{144}} = -\frac{1}{12} \pm \frac{5}{12}$$

$$x_1 = \frac{4}{12} = \frac{1}{3} \quad \vee \quad x_2 = -\frac{6}{12} = -\frac{1}{2}$$

c) $3x^2 + 6x - 9 = 3\cdot(x^2 + 2x - 3) = 3\cdot(x+3)\cdot(x-1)$

$$x_1 = -3 \quad \vee x_2 = 1$$

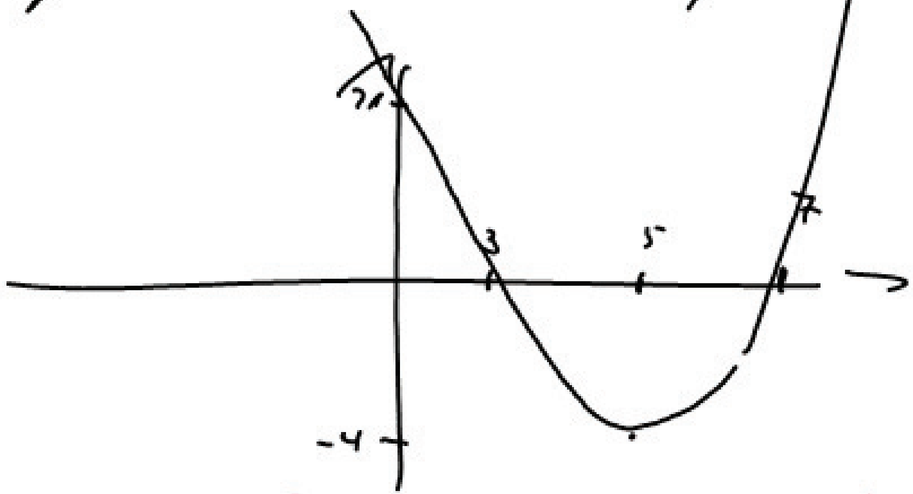
9) a) $f(x) = x^2 - 10x + 21 = (x-5)^2 - 25 + 21$
$= (x-5)^2 - 4 \Rightarrow S(5 / -4)$

$f(x) = 0 \Rightarrow (x-5)^2 - 4 = 0$
$(x-5)^2 = 4 \Rightarrow x - 5 = \pm\sqrt{4} = \pm 2$
$x_1 = 7 \vee x_2 = 3$

$S_y: f(0) = 21$ $\qquad$ $\mathbb{W} = y \in [-4; \infty[$

$\mathbb{W} = y \in \ldots$

$\mathbb{D} = x \in \ldots$



$(x+a)^2 + b \rightarrow S(-a / b)$

5) $g(x) = -x^2 - 2x + 15 = -(x^2 + 2x - 15)$

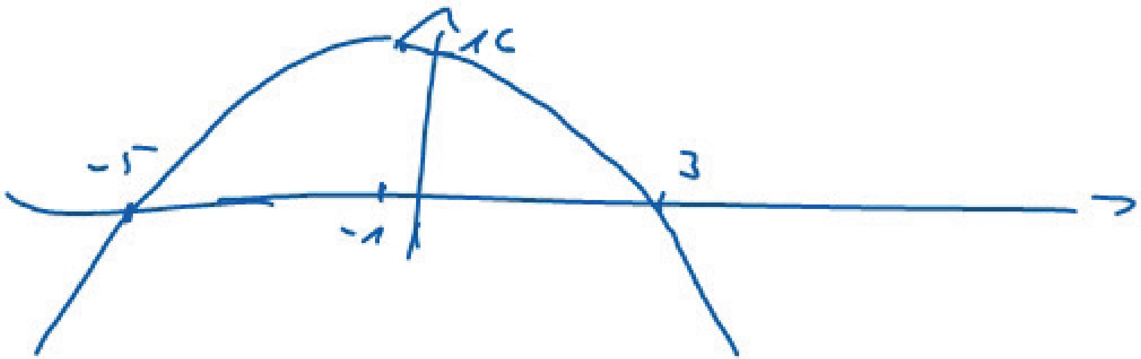
$= -(x+5)(x-3) = 0$

$x_1 = -5 \lor x_2 = 3$; $S_y(0 \mid 15)$

$S(-1 / f(-1)) = (-1 \mid 16)$

$f(-1) = -(4) \cdot (-4) = 16$

$W = y \in [16 ; -\infty[$



10) a) $|2x-6| \geq 5$

F	$x \geq 3$	$x < 3$
R	$2x-6 \geq 5$ $2x \geq 11$ $x \geq 11/2$	$-(2x-6) \geq 5$ $-2x+6 \geq 5$ $-2x \geq -1$ $x \leq 1/2$
E	$x \geq 5{,}5$	$x \leq 0{,}5$
P	$x=6: \ \|12-6\| \geq 5$ ✓	$x=0 \quad \|0-6\| \geq 5$ ✓

$\mathbb{L} = \{ x \in \mathbb{R} \mid x \leq 0{,}5 \ \vee \ x \geq 5{,}5 \}$

b) $\dfrac{x-2}{3+x} \leq 1 \quad | \cdot (3+x)$

$\boxed{x > -3}$	$x < -3$
$x - 2 \leq 3 + x$	$x - 2 \geq 3 + x \quad \vert -x$
$-2 \leq 3 \quad \checkmark$	$-2 \geq 3 \quad ↯$

$x = 0 \quad -\frac{2}{3} \leq 1 \quad \checkmark$

$$\mathbb{L} = \{ x \in \mathbb{R} \mid x > -3 \}$$

c) $x^2 - 10x \geq 16 \quad \Leftrightarrow \quad x^2 - 10x - 16 \geq 0$

$x_{1,2} = 5 \pm \sqrt{25 + 16} = 5 \pm \sqrt{41}$

$$\mathbb{L} = \{ x \in \mathbb{R} \mid x \geq 5 + \sqrt{41} \vee x \leq 5 - \sqrt{41} \},$$

da die Parabel nach oben geöffnet ist.

$x-3$

$$\lim_{x\to 3}\frac{6x-18}{(2x-5)-\sqrt{4\cdot x-11}}$$

$$\lim_{x\to 3}\frac{6(x-3)\cdot\left[(2x-5)+\sqrt{4x-11}\right]}{\underbrace{(2x-5)^2-(4x-11)}}$$

$$4x^2-20x+25-4x+11 \Rightarrow 4x^2-24x+36$$

$$(x-3)(4x-12) \qquad \Rightarrow 4x^2-12x-12x+36$$

$$\lim_{x\to 3}\frac{6\cdot\left[(2x-5)+\sqrt{4x-11}\right]}{4x-12}=\frac{6\cdot 2}{0}=\frac{12}{0}=\infty$$

11) a) -1 : $\lim_{x\to\infty}\left(\frac{3}{x}-\left(1+\frac{3}{x}\right)^3\right)=\left[0-(1+0)^3\right]=-1$

b)

$$\lim_{x\to 2}\left[\frac{\cos(5\pi-x\cdot\pi)}{4x-8}\right]=\frac{-1}{0}\Rightarrow \pm\infty$$

$$\lim_{x\to 2}\left[\frac{\cos(5\pi-\frac{x}{4}\cdot\pi)}{4x-8}\right]=\frac{0}{0}\Rightarrow \text{l'Hospital}$$

$$\lim_{x\to 2}\left[\frac{-\sin(5\pi-\frac{x}{4}\pi)\cdot(-\frac{\pi}{4})}{4}\right]=\frac{\pi/4}{4}=\frac{\pi}{16}$$

c) $\lim_{x\to\infty} e^{1/x}\cdot\frac{x^2\cdot(3-5/x+7/x^2)}{x^2(1/x-8/x^2+6)}=1\cdot\frac{1}{2}=\frac{1}{2}$

$(x+2)$

$$\lim_{x \to -2} \frac{4x+8}{(5+2x) - \sqrt{3x+7}} \cdot \frac{(5+2x) + \sqrt{3x+7}}{(5+2x) + \sqrt{3x+7}}$$

$$\lim_{x \to -2} \frac{4 \cdot (x+2) \cdot \left[(5+2x) + \sqrt{3x+7} \right]}{\underbrace{(5+2x)^2 - (3x+7)}}$$

$$25 + 20x + 4x^2 - 3x - 7 = \underbrace{4x^2 + 17x + 18}_{(x+2)(4x+9)}$$

$$\lim_{x \to -2} \frac{4 \cdot \left[(5+2x) + \sqrt{3x+7} \right]}{4x+9} = \frac{4 \cdot \left[(1) + \sqrt{1} \right]}{1}$$

$$= 8$$

$$\lim_{x\to -2} \frac{4x+8}{(5+2x)-\sqrt{3x+7}}$$

$$\lim_{x\to -2} \frac{4}{2-\frac{1}{2\cdot\sqrt{3x+7}}\cdot 3} = \left[\frac{4}{\underbrace{2-\frac{3}{2}}_{1/2}}\right] = 8$$

$$\lim_{x\to -1} \frac{7}{7+\frac{2\cdot 1}{2\cdot\sqrt{1-3x}}\cdot(-3)} = \left[\frac{7}{7-\frac{3}{2}}\right] = \ldots$$

$$= \frac{7}{\frac{14}{2}-\frac{3}{2}} = \frac{7}{11/2} = 7\cdot\frac{2}{11} = \frac{14}{11}$$

Sin 

13) $f(x) = \begin{cases} x^2 + ax + b & ; x \geq 1 \\ 2x - bx + 2a & ; x < 1 \end{cases}$

$f'(x) = \begin{cases} 2x + a & ; x \geq 1 \\ 2 - b & ; x < 1 \end{cases}$

<u>stetig:</u> $\lim\limits_{x \to 1^+} f(x) = f(1) = \lim\limits_{x \to 1^-} f(x)$

$$1 + a + b = 2 - b + 2a$$
$$2b - a = 1$$

$2b - (-b) = 1$

$3b = 1$

$b = 1/3$

<u>differenz.:</u> $\lim\limits_{x \to 1^+} f'(x) = f'(1) = \lim\limits_{x \to 1^-} f'(x)$

$$2 + a = 2 - b \quad \Rightarrow \quad a = -b$$

$a = -1/3$

14) a) $f(x) = 2 \cdot \underbrace{\sin^3(3x + 2\pi)} - 4$

$$\sin(3x) \cdot \underbrace{\cos(2\pi)}_{1} + \cos(3x) \cdot \underbrace{\sin(2\pi)}_{0}$$

$$f(x) = 2 \cdot \sin^3(3x) - 4$$

W: $\quad 2 \cdot [-1; 1] - 4 = [-2; 2] - 4 \Rightarrow y \in [-6; -2]$

Periode: $\quad P_{neu} = \frac{2\pi}{3} \Rightarrow f(x) = f(x + \frac{2}{3}\pi)$

$$2 \cdot \sin^3(3x) - 4 = 2 \cdot \sin^3(3 \cdot (x + \tfrac{2}{3}\pi)) - 4 \quad | +4 \cdot \tfrac{1}{2}$$

$$\sin^3(3x) = \sin^3(3x + 2\pi)$$

$$= [\sin(3x) \cdot \underbrace{\cos(2\pi)}_{1} + \cos(3x) \cdot \underbrace{\sin(2\pi)}_{0}]^3$$

$$= \sin^3(3x) \quad \checkmark$$

Symmetrie: $f(x) = -f(-x)$ $\qquad \sin(-x) = -\sin(x)$

$$2 \cdot \sin^3(3x) - 4 = -\left[2 \cdot \sin^3(-3x) - 4\right]$$
$$2 \cdot \sin^3(3x) - 4 = -2 \cdot \sin^3(-3x) + 4$$
$$2 \cdot \sin^3(3x) - 4 = 2 \cdot \sin^3(3x) + 4$$
$$-4 = 4$$

$f(x)$ ist um 4 nach unten verschoben

$\Rightarrow$ Punktsymmetrie zu $(0/\underline{-4})$

$$f(x) + 4 = -\left[f(-x) + 4\right]$$

$$\left(2 \cdot \sin^3(3x) - 4\right) \underline{+4} = -\left[\left(2 \cdot \sin^3(-3x) - 4\right) \underline{+4}\right]$$
$$2 \cdot \sin^3(3x) = -2 \cdot \sin^3(-3x)$$
$$= 2 \cdot \sin^3(3x)$$

17)

$f(x) = 1/2x^3 - 3x^2 + 9/2 x + 42$

$f'(x) = 3/2x^2 - 6x + 9/2$

$f''(x) = 3x - 6$

Extrempunkte: $f'(x) = 0 = 3/2x^2 - 6x + 9/2 \quad | \cdot 2/3$

$x^2 - 4x + 3 = (x-3)(x-1) = 0$

$x_1 = 1 \lor x_2 = 3$

$x_1 = 1 \quad f''(1) = -3 < 0 \Rightarrow HP(1 \mid f(1))$

$f(1) = 1/2 - 3 + 9/2 + 42 \Rightarrow HP(1 \mid 44)$

$x_2 = 3 \quad f''(3) = 3 > 0 \Rightarrow TP(3 \mid f(3))$

$f(3) = \frac{27}{2} - 27 + \frac{27}{2} + 42 \Rightarrow TP(3 \mid 42)$

$f''(x) = 0 = 3x - 6 \quad \Rightarrow x_w = 2 \quad \Rightarrow WP(2/43)$

$f(2) = \frac{1}{2}\cdot 8 - 12 + 9 + 42 = 43$

$f'(2) = 6 - 12 + \frac{9}{2} = -\frac{3}{2}$ $\qquad y = m\cdot x + s$

$43 = -1,5 \cdot 2 + s \qquad s = 46$

$y = 1,5\cdot x + 46$

Wie heißt die Senkrechte (Orthogonale)?

$m_1 \cdot m_2 = -1 \quad \Rightarrow \quad m_2 = +\frac{2}{3}$

$43 = +\frac{2}{3}\cdot 2 + s \qquad s = 41\frac{2}{3}$

$y = \frac{2}{3}x + 41\frac{2}{3}$

$f(x) = x^2 - 5x + 6$

Berechnen Sie den Schnittpunkt der beiden Tangenten der Nullstellen.

$f(x) = (x-2)(x-3) = 0 \qquad x_1 = 2 \lor x_2 = 3$

$f'(x) = 2x - 5$

$f'(2) = -1 \quad \Rightarrow t_1: \quad 0 = -1 \cdot 2 + b \quad \Rightarrow b = 2$

$y_1 = -x + 2$

$f'(3) = 1 \quad \Rightarrow t_2: \quad 0 = 1 \cdot 3 + b \quad \Rightarrow b = -3$

$y_2 = x - 3$

$t_1 = t_2 \qquad -x + 2 = x - 3 \iff 2x = 5 \iff \underline{\underline{x = 5/2}}$

$y_1 = -5/2 + 2 = -1/2 \qquad S(5/2 \,/\, -1/2)$

19) a) $f(x) = x^2 - 3x + 2 = (x-2)(x-1) = 0$

$$\int_1^2 f(x)\,dx = |F(2) - F(1)|$$

$$F(x) = \tfrac{1}{3}x^3 - \tfrac{3}{2}x^2 + 2x = \tfrac{1}{6}(2x^3 - 9x^2 + 12x)$$

$$F(2) = \tfrac{1}{6}\cdot(16 - 36 + 24) = \tfrac{2}{3}$$

$$F(1) = \tfrac{1}{6}\cdot(2 - 9 + 12) = \tfrac{5}{6}$$

$$|F(2) - F(1)| = |\tfrac{4}{6} - \tfrac{5}{6}| = |-\tfrac{1}{6}| = \tfrac{1}{6} \text{ FE}$$

5)

$$f(x) = 2 \cdot \sqrt{2x+5} \quad \wedge \; g(x) = x$$

$$f(x) = g(x) \quad \Rightarrow \quad 2 \cdot \sqrt{2x+5} = x \quad |^2$$

$$4 \cdot (2x+5) = 8x + 20 = x^2$$

$$x^2 - 8x - 20 = (x-10)(x+2) = 0 \qquad x_1 = -2$$

$$x_2 = 10$$

$$d(x) = x - 2 \cdot \sqrt{2x+5} \quad \Rightarrow \quad G(x) = (2x+5)^{3/2}$$

$$g(x) = 3/2 \cdot (2x+5)^{1/2} \cdot 2$$

$$= \underline{3} \, (2x+5)^{1/2}$$

$$\int_{-2}^{10} d(x)\, dx = D(10) - D(-2)$$

$$D(x) = 1/2\, x^2 - 2/3 \cdot (2x+5)^{3/2}$$

$$D(x) = \frac{x^2}{2} - \frac{2}{3} \cdot \sqrt[2]{(2x+5)^3}$$

$$D(10) = \frac{100}{2} - \frac{2}{3} \cdot \sqrt{(25)^3}$$

$$= 50 - \frac{250}{3} = -\frac{100}{3} = -33\tfrac{1}{3}$$

$$D(-2) = \frac{4}{2} - \frac{2}{3} \cdot \sqrt{1^3} = \frac{4}{3} = 1\tfrac{1}{3}$$

$$|D(10) - D(-2)| = |-33\tfrac{1}{3} - 1\tfrac{1}{3}| = |-34\tfrac{2}{3}| = 34\tfrac{2}{3} \quad FE$$

$$f(x) = 5 \cdot \sqrt[3]{4x-7} = 5 \cdot (4x-7)^{1/3}$$

$$G(x) = (4x-7)^{4/3}$$

$$g(x) = 4/3 \cdot (4x-7)^{1/3} \cdot 4 = 16/3 \cdot (4x-7)^{1/3}$$

$$F(x) = 15/16 \cdot (4x-7)^{4/3}$$