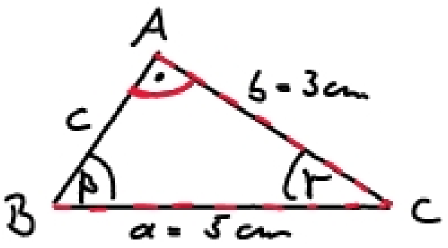
S 227

III a)



$a^2 = b^2 + c^2$

$25 = 9 + c^2 \quad | -9$

$16 = c^2 \quad | \sqrt{\ }$

$c = \pm\sqrt{16} = \pm 4 \quad \Rightarrow \mathbb{L} = \{4\}$

$\sin(\beta) = b/a = 3/5$

$\beta = \arcsin(3/5) = 36{,}87^\circ$

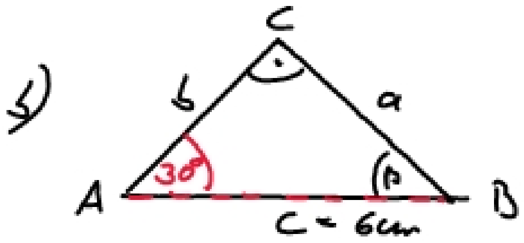
$\cos(\beta) = c/a = 4/5$

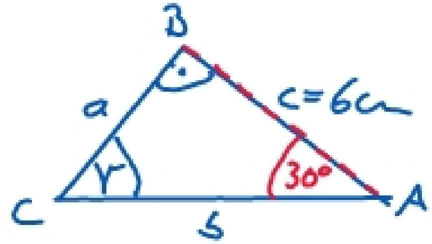
$\beta = \arccos(4/5) = 36{,}87^\circ$

$\sin(\gamma) = c/a = 4/5$

$\gamma = \arcsin(4/5) = 53{,}13^\circ$

b)





$\beta = (180^\circ - 90^\circ - 30^\circ) = 60^\circ$

$\gamma = 60^\circ$

$\sin(30^\circ) = a/6 \iff a = 6 \cdot \sin(30)^\circ = 3$

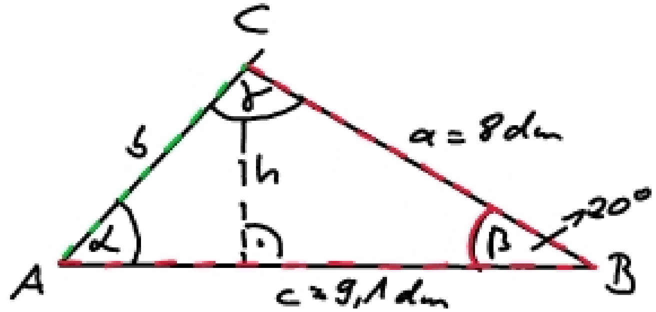
$\cos(30^\circ) = b/6 \iff b = 6 \cdot \cos(30^\circ) = 5{,}2$

$\cos(30^\circ) = \frac{6}{b} \iff b = \frac{6}{\cos(30^\circ)} = 6{,}93$

$\sin(30^\circ) = a/6{,}93 \iff a = 6{,}93 \cdot \sin(30^\circ)$

$a = 3{,}46$

IV



Fläche: $A = \frac{1}{2} \cdot c \cdot h$

$\sin(\beta) = h/a$

$h = a \cdot \sin(\beta)$

$h = 8 \cdot \sin(20°) = 2{,}74$

cos: $b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$

$b^2 = 8^2 + 9{,}1^2 - 2 \cdot 8 \cdot 9{,}1 \cdot \cos(20°)$

$b^2 = 10 \quad \Rightarrow \quad b = \sqrt{10} = \underline{3{,}16}$

$A = \frac{1}{2} \cdot 9{,}1 \cdot 2{,}74 = 12{,}47 dm^2$

$U = 8 + 9{,}1 + 3{,}16 = 20{,}26\ dm$

sin: $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \Leftrightarrow \sin(\alpha) = a/b \cdot \sin(\beta)$

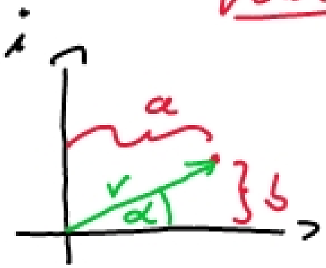
$\sin(\alpha) = \frac{8}{3{,}16} \cdot \sin(20°) = 0{,}865$

$\alpha = \arcsin(0{,}865) = 60°$

Σ: $\gamma = 180 - (60° + 20°) = 100°$

Vektoren

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix} = (a_1, a_2, \dots, a_n)^T$$



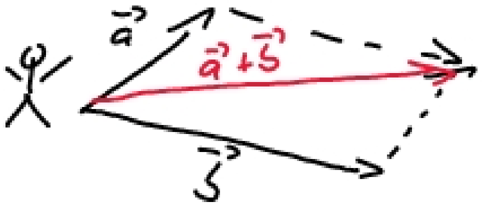
$$\vec{x} \in \mathbb{R}^3 \rightarrow \vec{x} = (x_1, x_2, x_3)^T$$ Vektor

$\alpha \in \mathbb{R}$ Skalar

$$\alpha \cdot \vec{x} = \alpha \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 8 \\ -2 \end{pmatrix} = -2 \cdot \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$$ "quasi gleich"

Linear abhängig



$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1+2 \\ -2+(-1) \\ 3+(-4) \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$$

Skalarprodukt $\Rightarrow$ Lösung ist ein Skalar

$$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} * \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = 1 \cdot (-2) + 2 \cdot 3 + (-3) \cdot 1 = 1$$

Vektorprodukt $\Rightarrow$ Lösung ist ein Vektor

$$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ -3 \\ 1 \\ 2 \\ -3 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \\ 1 \\ -2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 - 3 \cdot (-3) \\ (-3) \cdot (-2) - 1 \cdot 1 \\ 1 \cdot 3 - (-2) \cdot 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \\ 7 \end{pmatrix} = \vec{n}$$

Stellungsvektor

$$\left[\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}\right] * \left[2 \cdot \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}\right]$$

$$\begin{pmatrix} \cancel{3} \\ 1 \\ -2 \\ 3 \\ 1 \\ \cancel{-2} \end{pmatrix} \begin{pmatrix} \cancel{1} \\ 4 \\ -1 \\ 1 \\ 4 \\ \cancel{-1} \end{pmatrix} = \begin{pmatrix} -1 - (-8) \\ -2 - (-3) \\ 12 - 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 1 \\ 11 \end{pmatrix} * \begin{pmatrix} 6 \\ -2 \\ 6 \end{pmatrix} = 42 - 2 + 66 = 106$$

Lineare Unabhängig

Linear-Kombination: $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$

$\alpha = \beta = \gamma = 0 \rightarrow$ Triviallösung $\Rightarrow$ lineare Unabhängigkeit

$\vec{a} = (1;2;-1)^T \quad ; \quad \vec{b} = (2;5;2)^T \quad ; \quad \vec{c} = (-1;-2;3)^T$

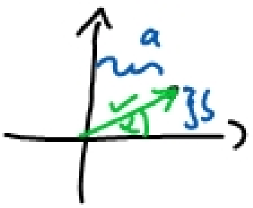
$$\left|\begin{array}{rrrl} \alpha & +2\beta & -\gamma & =0 \\ 2\alpha & +5\beta & -2\gamma & =0 \\ -\alpha & +2\beta & +3\gamma & =0 \end{array}\right| \begin{array}{l} \cdot(-2)\,+ \\ \\ + \end{array}$$

$$\left|\begin{array}{rrrl} \alpha & +2\beta & -\gamma & =0 \\ 0 & \beta & -4\gamma & =0 \\ 0 & 4\beta & 2\gamma & =0 \end{array}\right| \cdot(-4)\,+$$

$$\left|\begin{array}{rrrl} \alpha & +2\beta & -\gamma & =0 \\ 0 & \beta & -4\gamma & =0 \\ 0 & 0 & 10\gamma & =0 \end{array}\right|$$

lineare Unabhängigkeit

$\gamma = \beta = \alpha = 0$ Triviallösung



Vektoren

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = (a_1, a_2, \dots, a_n)^T$$

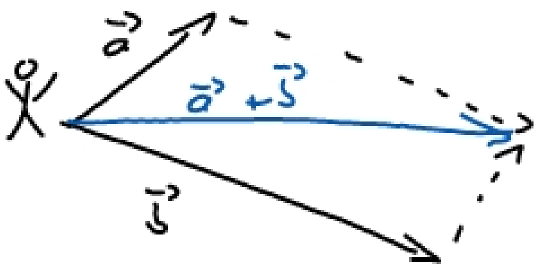
$$\vec{x} \in \mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \Rightarrow \vec{x} = (x_1, x_2, x_3)^T$$ Vektor

$\alpha \hat{=}$ Skalare

$$\alpha \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha \cdot x_1 \\ \alpha \cdot x_2 \\ \alpha \cdot x_3 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -8 \\ 6 \end{pmatrix} = -2 \cdot \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$$

linear abhängig (=)



$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$$

Skalarprodukt → ein Skalar kommt raus.

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} * \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 1 \cdot 3 + 2 \cdot (-1) + (-1) \cdot 2 = -1 \in \mathbb{R}$$

Vektorprodukt → ein Vektor kommt raus

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

Stellungsvektor

$\uparrow$ $\vec{n}$

$$\left|\begin{matrix} 1 \\ 2 \\ -1 \\ 1 \\ 2 \\ -1 \end{matrix}\right| \left|\begin{matrix} 3 \\ -1 \\ 2 \\ 3 \\ -1 \\ 2 \end{matrix}\right| = \begin{pmatrix} 2 \cdot 2 - (-1) \cdot (-1) \\ -1 \cdot 3 - 2 \cdot 1 \\ 1 \cdot (-1) - 3 \cdot 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ -7 \end{pmatrix}$$

$$\left[\begin{pmatrix}3\\-1\\2\end{pmatrix} \times \begin{pmatrix}1\\2\\1\end{pmatrix}\right] * \left(2\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}\right) = 24$$

$$\begin{pmatrix}3 & 1\\-1 & 2\\2 & 1\\3 & 1\\-1 & 2\\2 & 1\end{pmatrix} = \begin{pmatrix}-1 - 4\\2 - 3\\6 - (-1)\end{pmatrix} = \begin{pmatrix}-5\\-1\\7\end{pmatrix}$$

$$\begin{pmatrix}-5\\-1\\7\end{pmatrix} * \begin{pmatrix}-2\\0\\2\end{pmatrix} = (10 + 0 + 14) = 24$$

Linearkombination $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$

$\alpha = \beta = \gamma = 0$ 'Triviallösung' $\Rightarrow$ linear unabhängig

$\vec{a} = (-1;2;1)^T$; $\vec{b} = (1;-3;2)^T$; $\vec{c} = (2;1;1)^T$

$$\left|\begin{array}{rrrcl} -\alpha & +\beta & +2\gamma & = & 0 \\ 2\alpha & -3\beta & +\gamma & = & 0 \\ \alpha & +2\beta & +\gamma & = & 0 \end{array}\right| \quad \cdot 2 \quad + \quad +$$

$$\left|\begin{array}{rrrcl} -\alpha & +\beta & +2\gamma & = & 0 \\ 0 & -\beta & +5\gamma & = & 0 \\ 0 & 3\beta & +3\gamma & = & 0 \end{array}\right| \quad \cdot 3 \quad +$$

linear unabhängig

Triviallösung

$$\left|\begin{array}{rrrcl} -\alpha & +\beta & +2\gamma & = & 0 \\ 0 & -\beta & +5\gamma & = & 0 \\ 0 & 0 & 18\gamma & = & 0 \end{array}\right| \Rightarrow \alpha = \beta = \gamma = 0$$

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1) $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$

$$\left|\begin{array}{l} 2\alpha + 5\beta - 2\gamma = 0 \\ \alpha - \beta + 2\gamma = 0 \\ 3\alpha + 3\gamma = 0 \end{array}\right| \begin{array}{l} 2\cdot(-\gamma) + 5\cdot\gamma - 2\gamma = \gamma = 0 \\ -\gamma - \beta + 2\gamma = \gamma - \beta = 0 \iff \beta = \gamma \\ \to 3\alpha + 3\gamma = 0 \iff 3\alpha = -3\gamma \iff \alpha = -\gamma. \end{array}$$

$\Rightarrow \alpha = \beta = \gamma = 0$ Triviallösung

$$\begin{array}{l} \text{Pivot} \end{array} \left|\begin{array}{l} 2\alpha + 5\beta - 2\gamma = 6 \\ \alpha - \beta + 2\gamma = 5 \\ 3\alpha + 3\gamma = -6 \end{array}\right| \begin{array}{l} \\ \cdot(-2) \quad |\cdot(-3) \\ \\ \end{array}$$

$$\begin{array}{l} \\ \\ \text{Pivot} \ (1/3) \end{array} \left|\begin{array}{l} \alpha - \beta + 2\gamma = 5 \\ 0 \quad 7\beta - 6\gamma = -4 \\ 0 \quad 3\beta - 3\gamma = -21 \end{array}\right| \begin{array}{l} \\ \\ |\cdot(-7/3) \end{array}$$

$$\overbrace{\left|\begin{array}{l} \alpha - \beta + 2\gamma = 5 \\ 0 \quad \beta - \gamma = -7 \\ 0 \quad 0 \quad \gamma = 45 \end{array}\right|}^{\begin{array}{l} \alpha = -47 \\ \beta = 38 \\ \gamma = 45 \end{array}}$$