S173

1) $\frac{1}{4} \lg (256x^8) - 2\cdot \lg \frac{\sqrt{9}}{x^2} - 0{,}5 \cdot \lg \frac{x^4}{9} = 1{,}5 \lg (9x^4) + 3 \lg \frac{1}{2x^3} + 4 \cdot \lg \sqrt{27x}$

$$\lg (256x^8)^{1/4} - \lg (3/x^2)^2 - \lg (x^4/9)^{1/2} = \lg (9x^4)^{3/2} + \lg (1/2x^3)^3 + \lg (\sqrt{27x})^4$$

$$\lg \frac{4x^2}{3^2/x^4 \quad x^2/3} = \lg 3^3 x^6 \cdot \frac{1}{2^3 x^9} \cdot 3^6 \cdot x^2 \qquad | \uparrow^{10^x}$$

$$\frac{4}{3} x^4 = \frac{3^9}{2^3} \cdot \frac{1}{x} \qquad | \cdot x \qquad | \cdot 3/4$$

$$x^5 = \frac{3^{10}}{2^5} \qquad | \sqrt[5]{}$$

$$x = \frac{3^2}{2} = \frac{9}{2} \qquad \mathbb{D} = \mathbb{R}^+ \qquad \mathbb{L} = \{ \tfrac{9}{2} \}$$

2) $6\cdot\ln\sqrt[3]{3} - 4\cdot\left(\ln\sqrt{\frac{\sqrt{2}}{x}} + \frac{1}{2}\cdot\ln\frac{9}{x}\right) = 2\cdot\ln\frac{\sqrt{x^3}}{3} - 0{,}25\cdot\ln\left(16x^8\right) + 3\ln\frac{8}{x^2}$

$$\ln\left(3^{1/3}\right)^6 - \ln\left(\frac{2^{1/4}}{x^{1/2}}\right)^4 - \ln\left(\frac{9}{x}\right)^2 = \ln\left(\frac{x^{3/2}}{3}\right)^2 - \ln\left(16x^8\right)^{1/4} + \ln\left(\frac{8}{x^2}\right)^3$$

$$\ln\frac{\cancel{3^2}}{\frac{2}{x^2}\;\frac{3^4}{x^2}}_{\;3^2} = \ln\frac{\cancel{x^3}/3^2\cdot 2^9/x^6\cdot x^3}{2x^2\cdot x} \quad \Big|\ \uparrow e^x$$

$$\frac{1}{2\cdot 3^2}\cdot x^4 = \frac{2^8}{3^2}\cdot\frac{1}{x^5} \qquad \Big|\cdot x^5\cdot 3^2\cdot 2$$

$$x^9 = 2^9 \qquad \Big|\sqrt[9]{}$$

$$x = 2 \qquad \mathbb{D} = \mathbb{R}^+ \qquad \mathbb{L} = \{2\}$$

3) $f(x) = -\frac{1}{3}\cdot\ln(x^2-6x-40) = -\frac{1}{3}\cdot\ln[\underbrace{(x-10)(x+4)}]$

$\mathbb{D} = \{x\in\mathbb{R} \mid x>10 \vee x<-4\}$

$\lim\limits_{x\to 10^+} f(x) = \lim\limits_{x\to -4^-} f(x) = [-\frac{1}{3}\ln(0^+)] = -\frac{1}{3}\cdot(-\infty) = \infty$

$\lim\limits_{x\to \pm\infty} f(x) = [-\frac{1}{3}\cdot\ln\infty] = -\frac{1}{3}\cdot\infty = -\infty$ $\qquad \mathbb{W} = y\in\mathbb{R}$

4) $f(x) = \log(\underbrace{\sqrt{2x+4}}_{x\geq -2} - 8) - 12$

$$
\begin{aligned}
\sqrt{2x+4} - 8 &= 0 & &\mid +8\\
\sqrt{2x+4} &= 8 & &\mid \uparrow^2\\
2x+4 &= 64 & &\mid -4\\
2x &= 60 & &\mid :2\\
x &= 30
\end{aligned}
$$

$\lim\limits_{x\to\infty} g(x) = [\log(\sqrt{\infty}-8) - 12] = \infty$

$\lim\limits_{x\to 30^+} g(x) = [\log(\sqrt{64^+}-8) - 12] = -\infty$

$\mathbb{W} = y\in\mathbb{R}$

$\mathbb{D} = x\in\mathbb{R}^{>30}$

5) $h(x) = \frac{3x}{\underbrace{\ln(15-3x)}}$ $\qquad 15-3x=0 \qquad x=5$

$\ln(1)=0 \Rightarrow 15-3x=1 \quad |+3x-1$

$14 = 3x \quad |:3$

$x = 14/3 = 4\frac{2}{3}$

$\mathbb{D} = x \in \mathbb{R}^{<5} \setminus \{4\tfrac{2}{3}\}$

$\lim\limits_{x \to -\infty} h(x) = \left[\frac{-\infty}{\infty}\right] = -\infty$

$(3x)' = 3$

$(\ln(15-3x))' = \frac{1}{15-3x} \cdot (-3) = \frac{1}{x-5}$

$\lim\limits_{x \to 5^-} h(x) = \left[\frac{15}{\ln(0)}\right] = 0^-$

$\lim\limits_{x \to 4\frac{2}{3}} h(x) = \left[\frac{14}{\ln(1)}\right] = \infty \qquad \Rightarrow \mathbb{W} = y \in \mathbb{R}$

$f(x) = x^2 + p\cdot x + q = (x+a)^2 + b \;\to\; S(-a|b)$

$(x^2 + 2\cdot a\cdot x + a^2)$

$f(x) = x^2 + 4x + 3$

$\quad\;\; \downarrow\sqrt{\ } \quad \downarrow\cdot\tfrac{1}{2}$

Quadratische Ergänzung

$= (x+2)^2 - 2^2 + 3 = (x+2)^2 - 1 \qquad \underbrace{S(-2|-1)}_{TP}$

$(x^2 + 4\cdot x + 2^2)$

$S_y \Rightarrow (0|f(0)) = (0|3)$

$S_x \Rightarrow f(x) = 0 = (x+2)^2 - 1 \quad |+1$

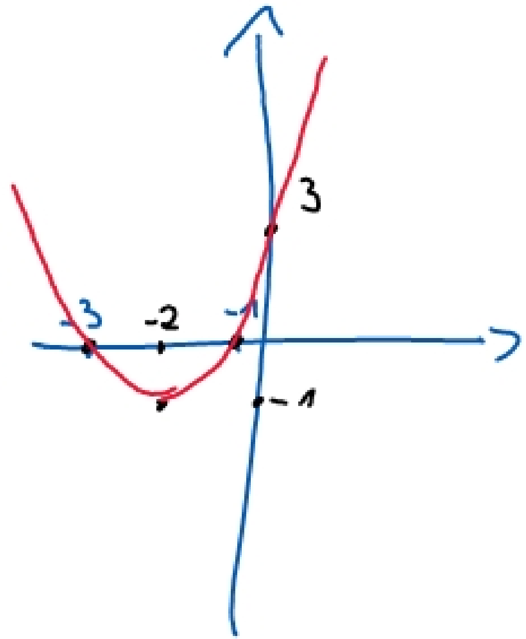
$1 = (x+2)^2 \quad |\sqrt{\ }$

$\pm 1 = x + 2 \quad |-2$

$x_1 = -1 \lor x_2 = -3$

$S_{x1} = (-1|0)$

$S_{x2} = (-3|0)$

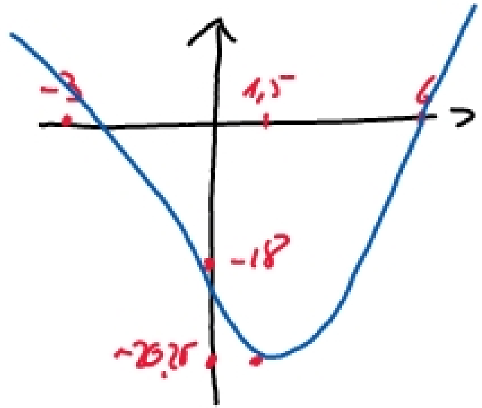


$$f(x) = x^2 - 3x - 18 = \left(x - \tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2 - 18$$

$$= \left(x - \tfrac{3}{2}\right)^2 - \tfrac{9}{4} - \tfrac{72}{4} = \underbrace{\left(x - \tfrac{3}{2}\right)^2 - \tfrac{81}{4}}$$

$S_y \Rightarrow (0 \mid -18)$

$TP \Leftarrow S(1{,}5 \mid -20{,}25)$



$$S_x: \quad \left(x - \tfrac{3}{2}\right)^2 - \tfrac{81}{4} = 0 \quad \mid + \tfrac{81}{4}$$

$$\left(x - \tfrac{3}{2}\right)^2 = \tfrac{81}{4} \quad \mid \sqrt{}$$

$$x - \tfrac{3}{2} = \pm \tfrac{9}{2} \quad \mid + \tfrac{3}{2}$$

$$x_1 = 6 \quad \vee \quad x_2 = -3$$

$$f(x) = x^2 - 3x - 18 = (x-6)(x+3)$$

$$x_1 = 6 \quad \vee \quad x_2 = -3$$

$$S(1{,}5 \mid f(1{,}5)) = S(1{,}5 \mid 4{,}5^2)$$

$$x^2 + \beta \cdot x + \gamma = 0 \qquad \beta, \gamma \in \mathbb{Q} \; ; \; x \in \mathbb{R}$$

$$\left(x + \beta/2\right)^2 - \left(\beta/2\right)^2 + \gamma = 0 \qquad \Big| + \left(\beta/2\right)^2 - \gamma$$

$$\left(x + \beta/2\right)^2 = \left(\beta/2\right)^2 - \gamma \qquad \Big| \sqrt{}$$

$$x + \beta/2 = \pm\sqrt{\left(\beta/2\right)^2 - \gamma} \qquad \Big| - \beta/2$$

$$x_{1/2} = -\frac{\beta}{2} \pm \sqrt{\left(\frac{\beta}{2}\right)^2 - \gamma} \qquad p = \beta \; ; \; q = \gamma$$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

S180

1) $3x^2 - 18x + 24 = 3 \cdot (x^2 - 6x + 8) = 3 \cdot (x-4)(x-2) = 0$

Satz v. Vieta : $\mathbb{L} = \{2;4\}$

2) $-0{,}5x^2 + 2x = -2{,}5 \quad | +2{,}5 \quad \cdot(-2)$

$x^2 - 4x - 5 = 0$ $\quad p = -4 \; ; \; q = -5$

p-q-Formel : $x_{1/2} = -\frac{(-4)}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 + 5} = 2 \pm \sqrt{9}$

$x_1 = 5 \lor x_2 = -1 \quad : \quad \mathbb{L} = \{-1; 5\}$

3) $x \cdot (2x - 20) = -32 \quad | +32$

$2x^2 - 20x + 32 = 2 \cdot (x^2 - 10x + 16) = 2 \cdot [(x-5)^2 - 25 + 16] = 0 \quad | \cdot \frac{1}{2}$

QE:

$(x-5)^2 - 9 = 0 \quad | +9$ $\qquad \mathbb{L} = \{2, 8\}$

$(x-5)^2 = 9 \quad | \sqrt{}$

$x - 5 = \pm\sqrt{9} = \pm 3 \quad | +5$ $\qquad x_1 = 8 \lor x_2 = 2$

4) $f(x) = -x^2 + 2x + 3 = -(x^2 - 2x - 3) = -(x-3)(x+1)$

Nach unten geöffnete Normalparabel $S_{x_1}(3/0)$ $S_{x_2}(-1/0)$

$S_y(0/3)$

Scheitelpunkt $S:(1/f(1)) = (1/4) \hat{=}$ HP

Achsensymmetrie zu $x = 1$

5) $g(x) = \frac{1}{4}x^2 + 2x + 3 = \frac{1}{4} \cdot (x^2 + 8x + 12) = \frac{1}{4} \cdot (x+6)(x+2)$

Nach oben geöffnete und um $\frac{1}{4}$ gestauchte Parabel

$S_{x_1}(-6/0)$ $S_{x_2}(-2/0)$ $S_y(0/3)$

Scheitelpunkt $S: (-4/f(-4)) = (-4/-1) \hat{=}$ TP

Achsensymmetrie zu $x = -4$

6) $h(x) = 100 - 4x^2 = -4 \cdot (x^2 - 25) = -4 \cdot (x-5)(x+5)$

Nach unten geöffnete um den Faktor 4 gestreckte Parabel

$S_{x_1}(5|0)$, $S_{x_2}(-5|0)$; $S_y(0|100)$ = Scheitelpunkt $\hat{=}$ HP

Achsensymmetrie zur y-Achse

7) $x^4 + 100 = 29x^2 \quad | -29x^2$

$x^4 - 29x^2 + 100 = (x^2 - 25)(x^2 - 4) = (x+5)(x-5)(x+2)(x-2)$

$\mathbb{L} = \{\pm 2; \pm 5\}$

8) $x^6 = 7x^3 + 8 \quad | -7x^3 - 8 \quad \Leftrightarrow \quad x^6 - 7x^3 - 8 = 0 \qquad z = x^3$

$z^2 - 7z - 8 = 0$

$z_{1/2} = \frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 + 8} = \frac{7}{2} \pm \sqrt{\frac{81}{4}} = \frac{7}{2} \pm \frac{9}{2} \qquad z_1 = 8, \; z_2 = -1 \qquad x = \sqrt[3]{z}$

$x_1 = \sqrt[3]{8} = 2 \;\vee\; x_2 = \sqrt[3]{(-1)} = -1 \quad \Rightarrow \quad \mathbb{L} = \{-1; 2\}$