S 165

1) $6\cdot \ln \sqrt[3]{e^2} - \frac{8}{e^{2\ln 0,5}} - \left(\frac{1}{2}e^{\ln 3^2} - \ln \frac{1}{\sqrt{e}}\right) + \frac{8}{\ln e^2} + e^{2\ln 3}$

$\ln(e^{2/3})^6 - 8e^{\ln(1/2)^{-2}} - \left(\frac{1}{2}\cdot 3^2 - \ln e^{-1/2}\right) + \frac{8}{2} + e^{\ln 3^2}$

$4 - 32 - \left(\frac{9}{2} + \frac{1}{2}\right) + 4 + 9 = -20$

2) $3\ln e^5 - 2\cdot\left(e^{2\ln 2} + \ln \frac{1}{\sqrt[4]{e}}\right) + \frac{10}{e^{\ln\sqrt{4}}} + 0,5\cdot e^{\ln 3}$

$\ln(e^5)^3 - 2\cdot\left(e^{\ln 2^2} + \ln e^{-1/4}\right) + 10/\sqrt{4} + \frac{1}{2}\cdot 3$

$15 - 2\cdot\left(4 - \frac{1}{4}\right) + 5 + \frac{3}{2} = 14$

3) $\frac{1}{6}\cdot \mathrm{ld}\, 2^3 + 3\cdot e^{2\ln 0,5} - \lg\sqrt{10} + 4\cdot\left(2^{4\,\mathrm{ld}\frac{1}{2}} - 8\cdot\ln\frac{1}{\sqrt{e}}\right) - 4\cdot 10^{\frac{1}{4}\lg 256}$

$\frac{1}{6}\cdot 3 + 3\cdot e^{\ln(1/2)^2} - \lg 10^{1/2} + 4\cdot\left(2^{\mathrm{ld}(1/2)^4} - 8\cdot\ln e^{-1/2}\right) - 4\cdot 10^{\lg(256)^{1/4}}$

$\frac{1}{2} + 3\cdot\frac{1}{4} - \frac{1}{2} + 4\cdot\left(\frac{1}{16} - 8\left(-\frac{1}{2}\right)\right) - 4\cdot 4$

$\frac{1}{2} + \frac{3}{4} - \frac{1}{2} + \frac{1}{4} + 16 - 16 = 1$

4) $\frac{2}{3}\cdot\left(\log 1000 - \frac{1}{2}\right) - \frac{2}{e^{\ln 0{,}5}} + 2^{3+\mathrm{ld}\,3} - (10^2)^{\log 3} + \ln\left(\frac{1}{\sqrt[3]{e}}\right)^2 - 4\,\mathrm{ld}\sqrt{2}$

$\frac{2}{3}\cdot\left(\log 10^3 - \frac{1}{2}\right) - \frac{2}{1/2} + 2^3\cdot 2^{\mathrm{ld}\,3} - 10^{\log 3^2} + \ln e^{-2/3} - \mathrm{ld}(2^{1/2})^4$

$2 - \frac{1}{3} - 4 + 8\cdot 3 - 9 - \frac{2}{3} - 2 = 10$

S169

I.1) $3\cdot\log x - 4\cdot\log \frac{2}{x} - \frac{1}{3}\cdot\log(x^2)^6 = \frac{2}{3}\lg 27 + \frac{1}{2}\lg x^4 - 2\log 6$

$\lg x^3 - \lg\left(\frac{2}{x}\right)^4 - \lg (x^{12})^{1/3} = \lg 27^{2/3} + \lg (x^4)^{1/2} - \lg 6^2$

$\lg \frac{x^3}{2^4/x^4 \cdot x^4} = \lg \frac{3^2\, x^2}{6^2}$ $\quad | \; 10^x$

$\frac{1}{16}x^3 = \frac{9}{36}x^2$ (green: $\frac{1}{4}$) $\quad | \; : x^2 \cdot 16$

$x = 4 \qquad \mathbb{D} = x \in \mathbb{R}^{>0}$

$\Rightarrow \mathbb{L} = \{4\}$

S 169

I.2) $3 \cdot \ln 4 - 0{,}5 \ln \frac{16}{x^4} + 2 \ln 8 = 1{,}5 \ln x^4 - 8 \ln \sqrt[4]{\frac{1}{x}} - 2 \cdot \ln \frac{1}{x}$

$\ln 4^3 - \ln \left(\frac{16}{x^4}\right)^{1/2} + \ln 8^2 = \ln (x^4)^{3/2} - \ln (x^{-1/4})^8 - \ln \left(\frac{1}{x}\right)^2$

$\ln \frac{4^3 \cdot 8^2}{4/x^2} = \ln \frac{x^6}{x^{-2} \cdot 1/x^2} \quad | \uparrow e^x$

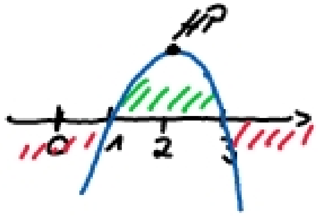
$4^2 \cdot 8^2 \cdot x^2 = 4^2 \cdot x^8 \quad | : 4^2 \ : x^2$

$\frac{4^2 \cdot 8^2}{4^2} = x^6 \quad \Leftrightarrow \quad 8^2 = (2^3)^2 = 2^6 = x^6$

$x = 2 \qquad \mathbb{D} = x \in \mathbb{R}^{>0} \qquad \mathbb{L} = \{2\}$

II. 3) $f(x) = \frac{2}{5} \cdot \ln (4x - 3 - x^2) = \frac{2}{5} \cdot \ln(-(x^2 - 4x + 3))$

$= \frac{2}{5} \cdot \ln(-(x-3)(x-1))$




$\lim_{x \to 1^+} f(x) = [\frac{2}{5} \cdot \ln(0^+)] = -\infty$

$x = 2: -(2-3)(2-1) > 0$

$x = 0: -(0-3)(0-1) < 0$

$x = 4: -(4-3)(4-1) < 0$

$f(2) = \frac{2}{5} \cdot \ln(1) = 0$

$\mathbb{W} = y \in \mathbb{R}^{\leq 0}$

$\mathbb{D} = x \in ]1;3[ = x \in (1;3)$

## Kettenregel

$f(x) = a \cdot x^n$
$f'(x) = a \cdot n \cdot x^{n-1}$

### Exponential

$f(x) = e^{\heartsuit} \rightarrow f'(x) = e^{\heartsuit} \cdot \heartsuit'$

$f(x) = e^{2-3x} \rightarrow f'(x) = e^{2-3x} \cdot (-3)$

$f(x) = 3 \cdot x^4$
$f'(x) = 3 \cdot 4 \cdot x^{4-1} = 12 \cdot x^3$

$f(x) = 2 \cdot \sqrt[3]{x^2} = 2 \cdot x^{2/3}$
$f'(x) = 2 \cdot 2/3 \cdot x^{2/3 - 1}$
$= 4/3 \cdot x^{-1/3} = 4/3 \cdot \frac{1}{\sqrt[3]{x}}$

### Potenz

$f(x) = (\heartsuit)^n \rightarrow n \cdot (\heartsuit)^{n-1} \cdot \heartsuit'$

$f(x) = (3x^2-4)^3 \rightarrow f'(x) = 3 \cdot (3x^2-4)^{3-1} \cdot (3x^2-4)' = 3 \cdot (3x^2-4)^2 \cdot 6x$

### Trigonom.

$f(x) = \sin(\heartsuit) \rightarrow f'(x) = \cos(\heartsuit) \cdot \heartsuit'$
$\cos(\heartsuit) \qquad -\sin(\heartsuit) \cdot \heartsuit'$

### Logarithmus

$f(x) = \ln(\heartsuit) \rightarrow f'(x) = \frac{1}{\heartsuit} \heartsuit'$

$f(x) = \ln(\sqrt{x}) \rightarrow f'(x) = \frac{1}{\sqrt{x}} \cdot (\sqrt{x})'$

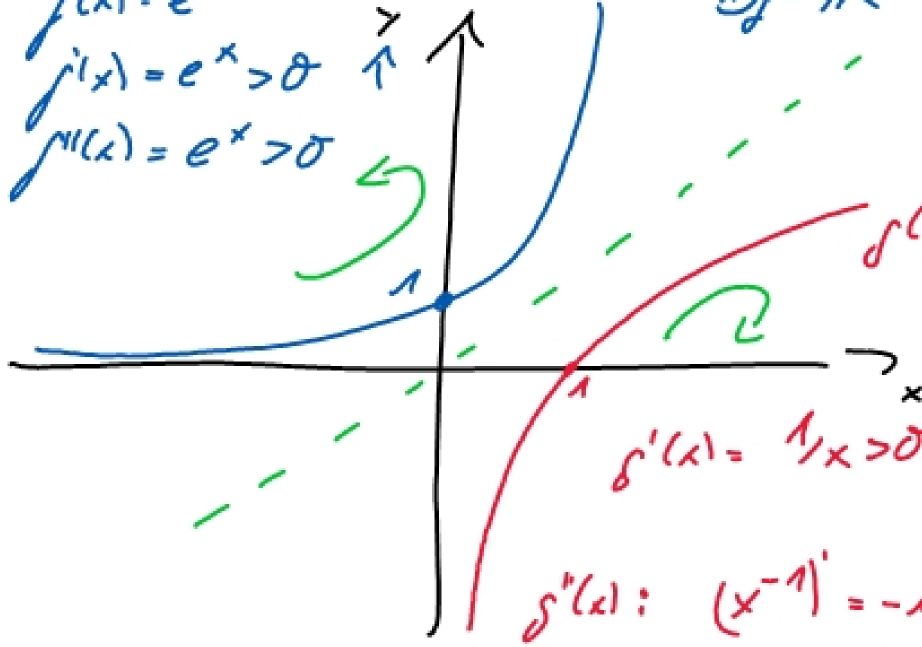
$\Rightarrow \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$

$f(x) = e^x$

$f'(x) = e^x > 0$ ↑

$f''(x) = e^x > 0$

$D_f = x \in \mathbb{R}$ ; $W_f = y \in \mathbb{R}^{>0}$




$f(x) = \ln(x)$

$D_f = x \in \mathbb{R}^{>0}$

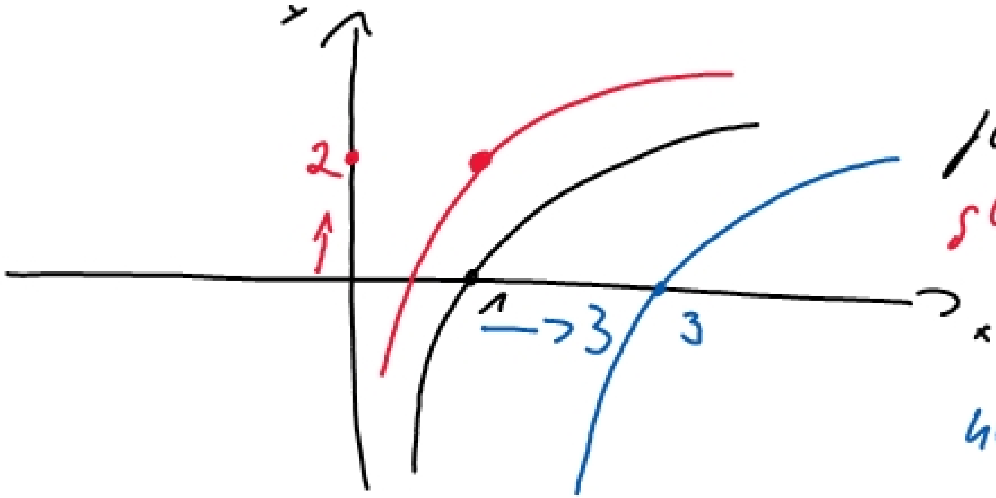
$f'(x) = 1/x > 0$ ↑

$W_f = x \in \mathbb{R}$

$f''(x): \ (x^{-1})' = -1 \cdot x^{-2} = -\frac{1}{x^2} < 0$

$\ln 0 = y$

$e^y = 0$




$f(x) = \ln(x)$

$g(x) = \ln(x) + 2$

$h(x) = \ln(x-2)$