Wachstums- / Zerfallsfunktion

$$A(x) = A_0 \cdot q^x$$

Wachstum : $q > 1$: $A_0 = 1000,-$; $p = 2\% \to q = 1{,}02$

$x \mathrel{\hat=}$ Jahre

jährlich : $A(x) = 1.000,- \cdot 1{,}02^x$

vierteljährlich : $A(x) = 1.000,- \cdot 1{,}02^{4x}$

$x \mathrel{\hat=}$ Monate

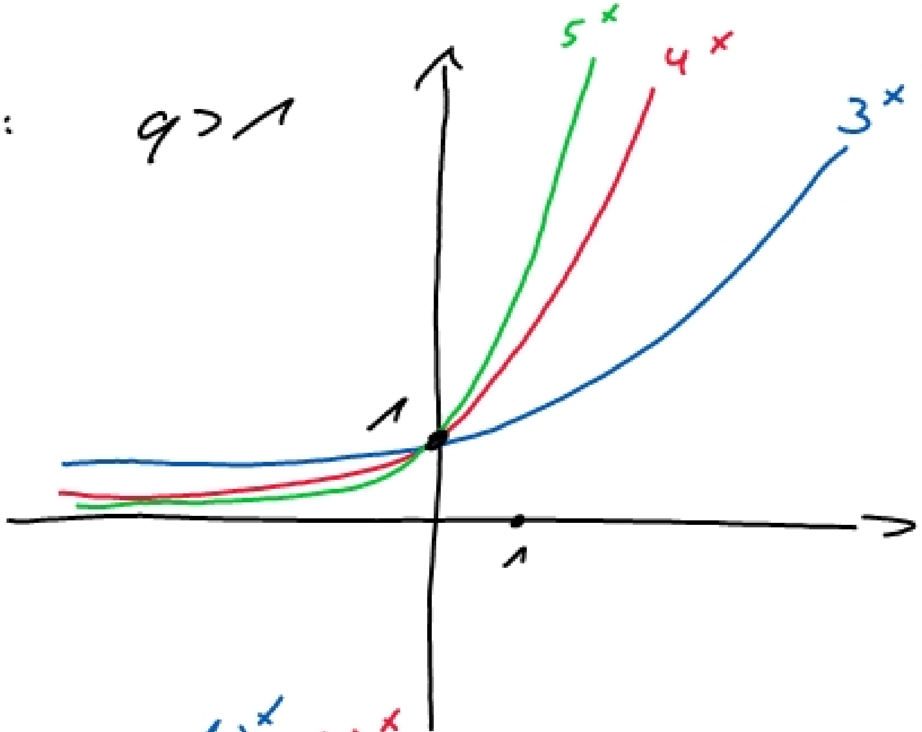
vierteljährlich $A(x) = 1.000,- \cdot 1{,}02^{\frac{1}{3}x}$

Zerfall : $q < 1$ 3 % Verlust pro Halbjahr

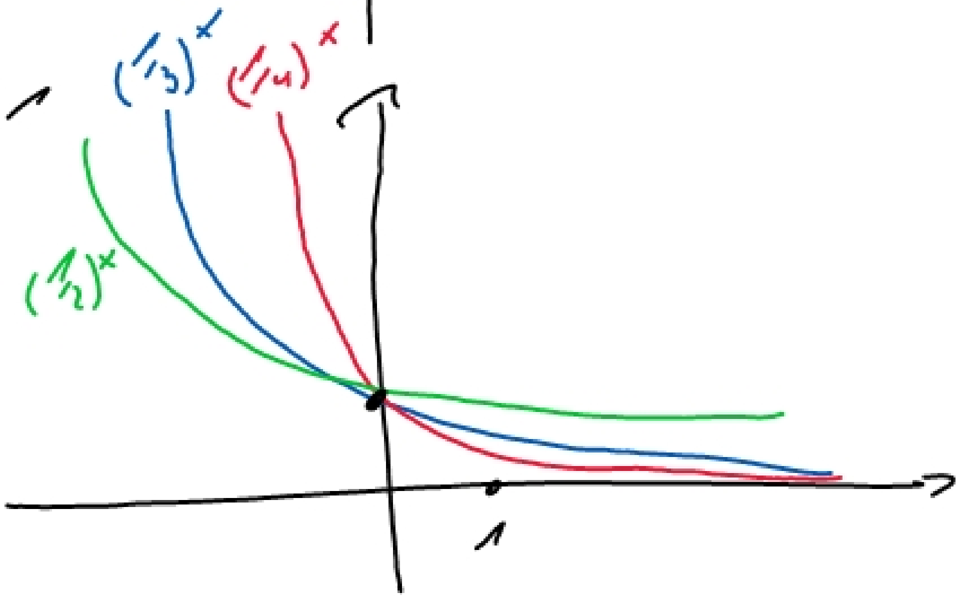
$x \mathrel{\hat=}$ Jahre : $A(x) = A_0 \cdot 0{,}97^{2x}$

Halbwertszeit 50 Jahre $A(x) = A_0 \cdot 0{,}5^{\frac{1}{50}x}$

Wachstum : $q > 1$



Zerfall : $q < 1$



1) $K_0 = 2.000,-$ $p = 2\%$ => $q = 1{,}02$ vierteljährlich

a) $K_n = 2.000 \cdot 1{,}02^{4 \cdot n}$ => $K_{10} = 2.000 \cdot 1{,}02^{40} = 4.416{,}08$

b) $1{,}02^{4n} = (1{,}02^4)^n = 1{,}082^n$ => $8{,}2\%$

c) $K_n = 9.750{,}88 = 2.000 \cdot 1{,}02^{4n}$ $| : 2000$

$4{,}875 = 1{,}02^{4n}$ $| \log$

$4n = \log_{1{,}02} 4{,}875 = 80$ $| : 4$

$n = 20$ Jahre

2) $A_0 = 1.000 L = 1.000 \, dm^3 = 1.000.000 \, cm^3$

$p = -5\% \Rightarrow q = 0{,}95$ wöchentlich

a) $f(x) = 1.000.000 \cdot 0{,}95^{52x} = 69.442{,}84 \, cm^3$

b) $0{,}95^{52x} < 0{,}5 \quad | \log$

$52x \cdot \log 0{,}95 < \log 0{,}5 \quad | : \log 0{,}95 \; (<0)$

$52x > \log 0{,}5 / \log 0{,}95 \quad | : 52$

$x > 0{,}2598 \quad | \cdot 365$

$x > 94{,}85 \Rightarrow 95$ Tage

3) $$5\cdot\log(2x) + 4\cdot\log\sqrt{0{,}5x} - \tfrac{1}{2}\log(16x^4) - 2\cdot\log(\tfrac{1}{4})$$

$$\log(2x)^5 + \log\left(\left(\tfrac{1}{2}x\right)^{1/2}\right)^4 - \log(16x^4)^{1/2} - \log\left(\tfrac{1}{4}\right)^2$$

$$\log\frac{2^5x^5 \quad \frac{1}{2^2}x^2}{4x^1 \quad \frac{1}{4^2}} = \log 2^5x^5 = 5\cdot\log(2x)$$

4) $$2\cdot\ln(3a^2) - 6\cdot\ln\sqrt[3]{2a^4} + \tfrac{1}{3}\ln\left[27(a^2)^6\right] - 4\cdot\ln\left(\tfrac{2}{a}\right)$$

$$\ln(3a^2)^2 - \ln\left((2a^4)^{1/3}\right)^6 + \ln\left(27a^{12}\right)^{1/3} - \ln\left(\tfrac{2}{a}\right)^4$$

$$\ln\frac{3^2a^4 \quad 3a^4}{2^2a^8 \quad 2^4/a^4} \quad \ln\frac{3^3a^4}{2^6} = \ln\frac{27}{64}\cdot a^4$$

$$f(x) = e^x \quad \to \quad f'(x) = e^x \cdot (x)' = e^x \cdot 1 = e^x$$

$$f(x) = e^{\heartsuit} \quad \to \quad f'(x) = e^{\heartsuit} \cdot \heartsuit'$$

$$f(x) = 42^x = \left(e^{\ln 42}\right)^x = \underline{e^{\ln 42 \cdot x}}$$

$$f'(x) = e^{\ln 42 \cdot x} \cdot (\ln 42 \cdot x)'$$

$$= \underline{e^{\ln 42 \cdot x}} \cdot \ln 42$$

$$= 42^x \cdot \ln 42$$

$$f(x) = a^x \quad \to \quad f'(x) = a^x \cdot \ln a$$

1) $\lg \frac{1}{100} - \sqrt{e}^{\ln 4} + 4^{\operatorname{ld} 3} - 2 \cdot \operatorname{ld} \frac{1}{4}$

$\lg 10^{-2} - e^{\frac{1}{2} \cdot \ln 4} + 2^{2 \operatorname{ld} 3} - 2 \operatorname{ld} 2^{-2}$

$-2 \quad - 2 \quad + 9 \quad + 4 \quad = 9$

2) $100^{\lg 3} - \ln \frac{1}{e^2} + \frac{1}{2} \cdot \operatorname{ld} 16 - e^{-3 \ln \frac{1}{2}}$

$10^{2 \cdot \lg 3} - \ln e^{-2} + \frac{1}{2} \cdot \operatorname{ld} 2^4 - e^{\ln (\frac{1}{2})^{-3}}$

$9 \quad - (-2) \quad + 2 \quad - 8 \quad = 5$