Wachstums- / Zerfallfunktion

$$A(x) = A_0 \cdot q^x$$

<u>Wachstum</u>: $1000,- = A_0$; $p = 2\,\% \to q = 1{,}02$

$x \mathrel{\hat{=}} \text{Jahre}$ jährlich: $A(x) = 1.000,- \cdot 1{,}02^{x}$

vierteljährlich: $A(x) = 1.000,- \cdot 1{,}02^{4x}$

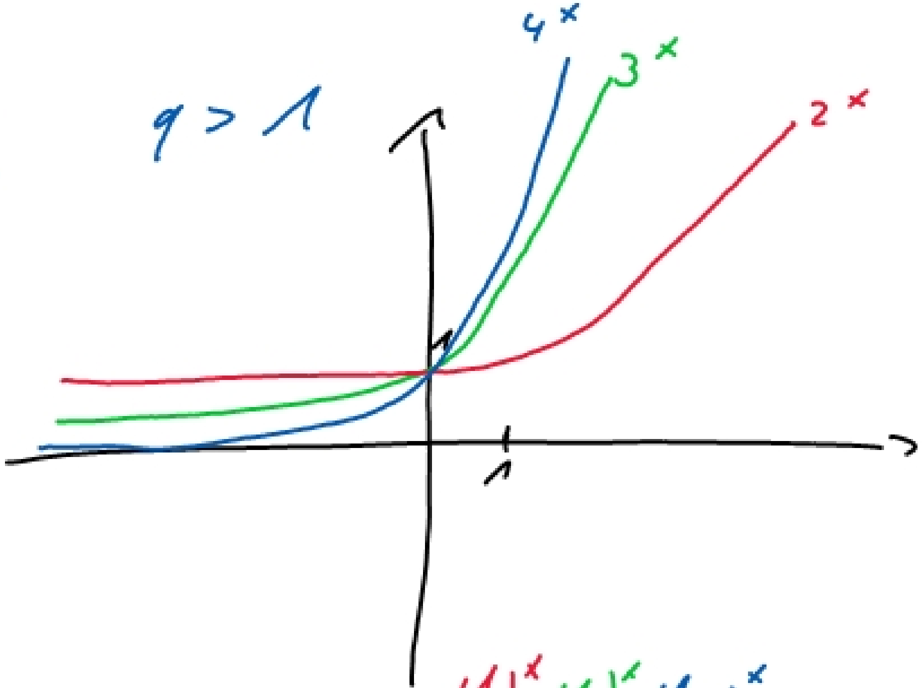
$x \mathrel{\hat{=}} \text{Monate}$ $\hookrightarrow$ $A(x) = 1.000,- \cdot 1{,}02^{1/3\,x}$

<u>Zerfall</u>: $3\,\%$ Verlust pro Halbjahr $x \mathrel{\hat{=}} \text{Jahre}$

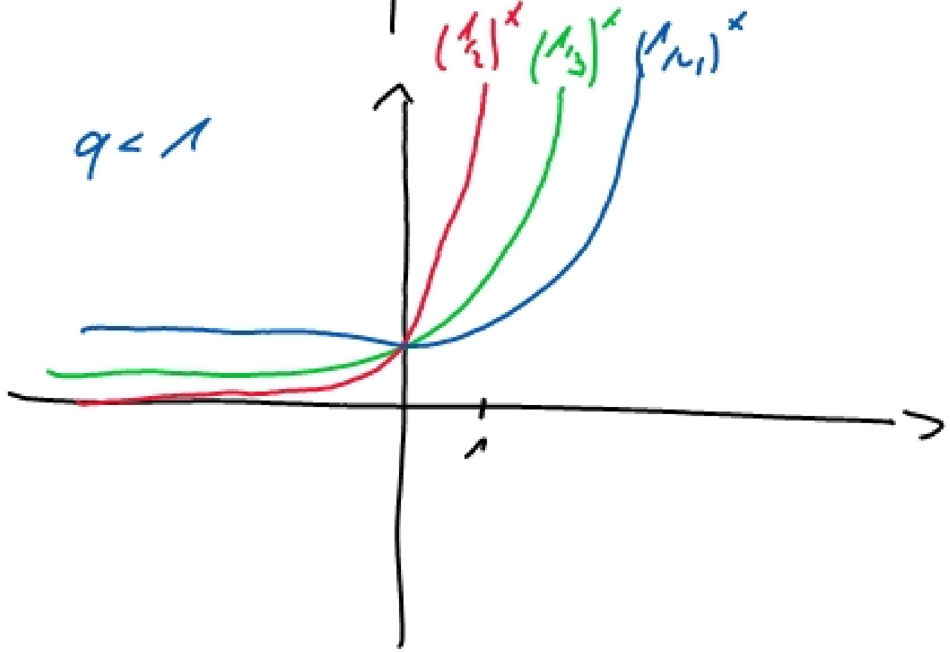
$$A(x) = A_0 \cdot 0{,}97^{2x}$$

Halbwertszeit 50 Jahren $A(x) = A_0 \cdot 0{,}5^{1/10\,x}$

Wachstumsgesetz $q > 1$



Zerfallsgesetz $q < 1$



1) $K_0 = 2.000$; $p = 2\%$ -> $q = 1{,}02$ vierteljährlich

a) $K_n = K_0 \cdot q^t = 2.000,- \cdot 1{,}02^{4 \cdot t}$

$= 2.000,- \cdot 1{,}02^{40} = 4.416{,}08$

b) $1{,}02^{4t} = (1{,}02^4)^t = 1{,}082^t$ => $p = 8{,}2\%$

c) $K_n = 9.750{,}88 = 2.000 \cdot 1{,}02^{4t}$ $\quad | : 2.000$

$4{,}875 = 1{,}02^{4x}$ $\quad | \log$

$4x = \log_{1{,}02} 4{,}875 = 80$ $\quad | : 4$

$x = 20$ Jahre

2) $A_0 = 1.000\,l = 1.000\,dm^3 = 10^6\,cm^3$

$p = -5\,\% \rightarrow q = 0{,}95$ wöchentlich

a) $A(x) = A_0 \cdot 0{,}95^{52 \cdot x} = 10^6 \cdot 0{,}95^{52} \approx 69.442{,}84\,cm^3$

b) $0{,}95^{52x} < 0{,}5 \quad | \log$

$52x \cdot \lg 0{,}95 < \lg 0{,}5 \quad | : \lg 0{,}95$

$52x > \lg 0{,}5 / \lg 0{,}95 \quad | : 52$

$x > 0{,}2598 \quad | \cdot 365$

$x > 94{,}85 \rightarrow$ nach 95 Tagen

3)
$$5\cdot\log(2x)+4\cdot\log\left(\sqrt{0{,}5x}\right)-\tfrac{1}{2}\cdot\log(16x^4)-2\cdot\log 0{,}25$$

$$\log(2x)^5+\log\left((0{,}5x)^{1/2}\right)^4-\log(16x^4)^{1/2}-\log\left(1/4\right)^2$$

$$\log\frac{2^5x^5\cdot 1/2^2\cdot x^2}{4x^2\cdot 1/16}=\log x^5\cdot 2^5=5\cdot\log(2x)$$

4)
$$2\cdot\ln(3a^2)-6\cdot\ln\sqrt[3]{2a^4}+\tfrac{1}{3}\cdot\ln\left(27(a^2)^6\right)-4\ln\left(\tfrac{2}{a}\right)$$

$$\ln(3a^2)^2-\ln\left((2a^4)^{1/3}\right)^6+\ln(27a^{12})^{1/3}-\ln\left(\tfrac{2}{a}\right)^4$$

$$\ln\frac{3^2a^4\quad 3\,a^4}{2^2a^8\quad 2^4/a^4}\;\ln\frac{3^3\;a^4}{2^6}=\ln\frac{27}{64}a^4$$

$$f(x) = e^x \longrightarrow f'(x) = e^x \cdot (x)' = e^x \cdot 1 = e^x$$

$$f(x) = e^{\heartsuit} \longrightarrow f'(x) = e^{\heartsuit} \cdot \heartsuit'$$

$$f(x) = 42^x = \left(e^{\ln 42}\right)^x = e^{\ln 42 \cdot x}$$

$$f'(x) = e^{\ln 42 \cdot x} \cdot (\ln 42 \cdot x)' = e^{\ln 42 \cdot x} \cdot \ln 42$$

$$= 42^x \cdot \ln(42)$$

$$f(x) = a^x \longrightarrow f'(x) = a^x \cdot \ln a$$

1) $\lg \frac{1}{100} - \sqrt{e}^{\ln 4} + 4^{\mathrm{ld}\, 3} - 2\,\mathrm{ld}\left(\frac{1}{4}\right)$

$\lg 10^{-2} - \left(e^{1/2}\right)^{\ln 4} + \left(2^{2}\right)^{\mathrm{ld}\, 3} - 2\,\mathrm{ld}\; 2^{-2}$

$\underline{-2} - e^{\frac{1}{2}\ln 4} + 2^{2\,\mathrm{ld}\, 3} \underline{-2(-2)} = -e^{\ln 4^{1/2}} + 2^{\mathrm{ld}\, 3^{2}} + 2$

$= -2 + 9 + 2 = 9$

2) $100^{\lg 3} - \ln \frac{1}{e^{2}} + \frac{1}{2}\,\mathrm{ld}\, 16 - e^{-3 \ln \frac{1}{2}}$

$10^{2 \lg 3} - \ln e^{-2} + \frac{1}{2}\,\mathrm{ld}\, 2^{4} - e^{\ln \left(\frac{1}{2}\right)^{-3}}$

$9 + 2 + 2 - 8 = 5$