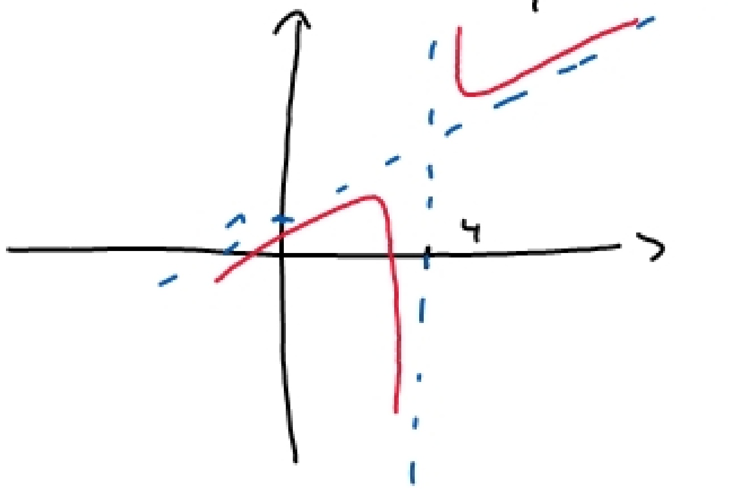
$$f(x) = \frac{x^2 - 3x + 5}{x - 4} \quad ; \quad D = \mathbb{R} \setminus \{4\}$$

$$\lim_{x \to \infty} f(x) = \frac{\cancel{x^2}\,(1 \underbrace{- \tfrac{3}{x} + \tfrac{5}{x^2}}_{\to 0})}{\cancel{x}\,(1 - \underbrace{\tfrac{4}{x}}_{\to 0})} = [x] = \infty$$

$$\begin{array}{l} (x^2 - 3x + 5) : (x - 4) = x + 1 + \frac{9}{x-4} \\ \underline{-(x^2 - 4x)} \\ \qquad\quad x + 5 \\ \qquad \underline{-(x - 4)} \\ \qquad\qquad 9 \end{array}$$

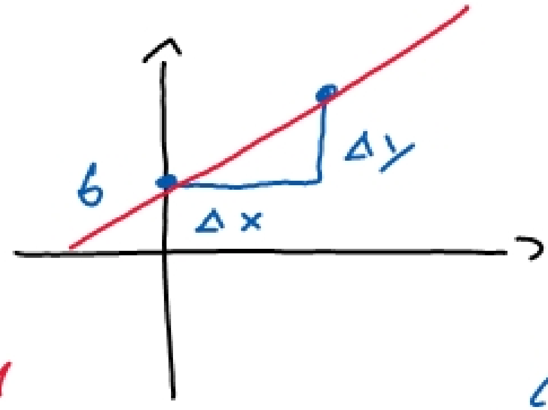



| | | |
|---|---|---|
| $\lim_{x \to \infty} f(x) = k$ | | waagerecht |
| $\lim_{x \to \infty} f(x) = \infty$ | | diagonal |
| $\lim_{x \to k} f(x) = \infty$ | | senkrecht |
| $\lim_{x \to k} f(x) = k$ | | behebbare Lücke |

$f(x) \Rightarrow$ y-Koordinate $\qquad P(x_1 | y_1) \quad Q(x_2 | y_2)$

$f'(x) \Rightarrow$ Steigung $\qquad m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$

$f''(x) \Rightarrow$ Krümmung $\begin{cases} >0 & \text{links} \\ <0 & \text{rechts} \end{cases}$



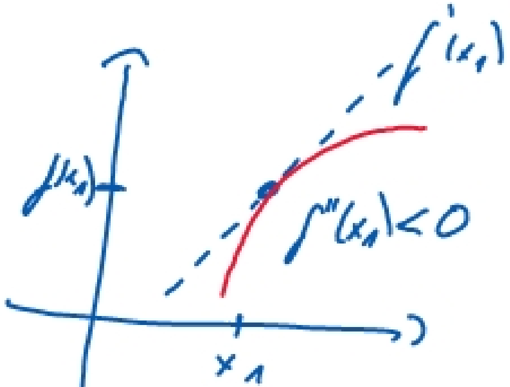

Wendetangente: $y = m \cdot x + b$

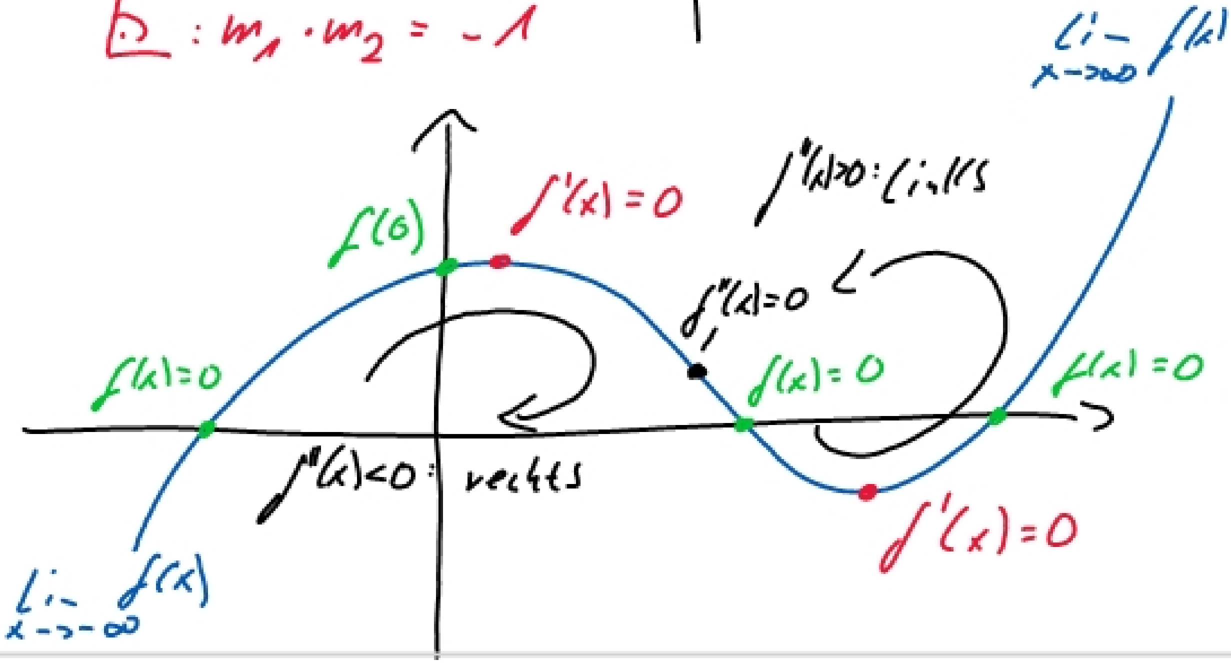
$f''(x) = 0 \rightarrow x_w$

$f(x_w) = y$

$f'(x_w) = m$

$\perp : m_1 \cdot m_2 = -1$



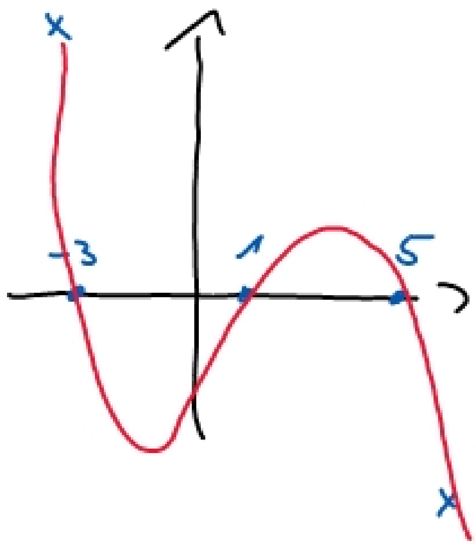
1) $f(x) = -x^3 + 3x^2 + 13x - 15$ → Nullstellen + Skizze

2) $\dfrac{2}{5x} - \dfrac{3}{4} + \dfrac{5}{12} - 1\frac{1}{6} = \dfrac{4}{15x} - 0{,}3$

3) $\dfrac{\frac{a}{3} + 2 + \frac{3}{a}}{\frac{1}{6} + \frac{1}{2a}}$   [ $x^4 - 4x^3 - 8x^2 + 29x + 12 = 0$ ]

$\rightarrow M_{15} = \{\pm 1; \pm 3; \pm 5; \pm 15\}$

1) $f(x) = -(x^3 - 3x^2 - 13x + 15)$   $x = 1: f(x) = 0 \rightarrow (x-1)$




$(x^3 - 3x^2 - 13x + 15) : (x - 1) = \underbrace{x^2 - 2x - 15}_{(x-5)(x+3)}$

$-(x^3 - x^2)$

$\quad / \quad -2x^2 - 13x + 15$

$\quad -(-2x^2 + 2x)$

$\quad\quad / \quad -15x + 15$

$\quad\quad -(-15x + 15)$

$\quad\quad\quad / \quad /$

$f(x) = -(x-1)(x-5)(x+3) = 0$

$L = \{-3; 1; 5\}$

2) $\frac{2}{5x} - \frac{3}{4} + \frac{5}{12} - \frac{7}{6} = \frac{4}{15x} - \frac{3}{10} \quad | \cdot HN \; (\cdot 60 \cdot x|$

$\frac{2 \cdot 60x}{5x} - \frac{3 \cdot 60x}{4} + \frac{5 \cdot 60x}{12} - \frac{7 \cdot 60x}{6} = \frac{4 \cdot 60x}{15 \cdot x} - \frac{3 \cdot 60x}{10}$

$24 - 45x + 25x - 70x = 16 - 18x \quad | +18x - 24$

$-72x = -8 \quad |:(-72$

$x = -8/-72 = 1/9 = 0{,}\bar{1}$

3) $\frac{\frac{a}{3} + 2 + \frac{3}{a}}{\frac{1}{6} + \frac{1}{2a}} = \frac{\frac{a^2+6a+9}{3a}}{\frac{a+3}{6a}}$ $\longrightarrow (a+3)^2$

$[(x+2)(x+1)(x-3)(x-4)]$

$\frac{(a+3)^{\cancel{2}}}{\underline{3a}} \cdot \frac{\overset{2}{\cancel{6a}}}{\underline{a+3}} = 2 \cdot (a+3) = 2a+6$

1) $$\left( \sqrt[3]{x^2} \cdot \sqrt[5]{1/x^{-3}} \cdot \sqrt{\sqrt[3]{1/x^4}} \right)^2$$

$$\left( (x^2)^{1/3} \cdot (x^3)^{1/5} \cdot \left((x^{-4})^{1/3}\right)^{1/2} \right)^2$$

$$\left( x^{2/3} \cdot x^{3/5} \cdot x^{-4/6} \right)^2$$

$$\left( x^{\frac{2}{3} + \frac{3}{5} - \frac{2}{3}} \right)^2 = \left( x^{\frac{10+9-10}{15}} \right)^2$$

$$= \left( x^{3/5} \right)^2 = x^{6/5} = \sqrt[5]{x^6}$$

$$x^{5/5} \cdot x^{1/5} = x \cdot \sqrt[5]{x}$$

$$\sqrt[3]{8^5} = (8^5)^{1/3}$$

$$= 8^{5 \cdot 1/3} = 8^{1/3 \cdot 5}$$

$$= \left(8^{1/3}\right)^5 = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$$