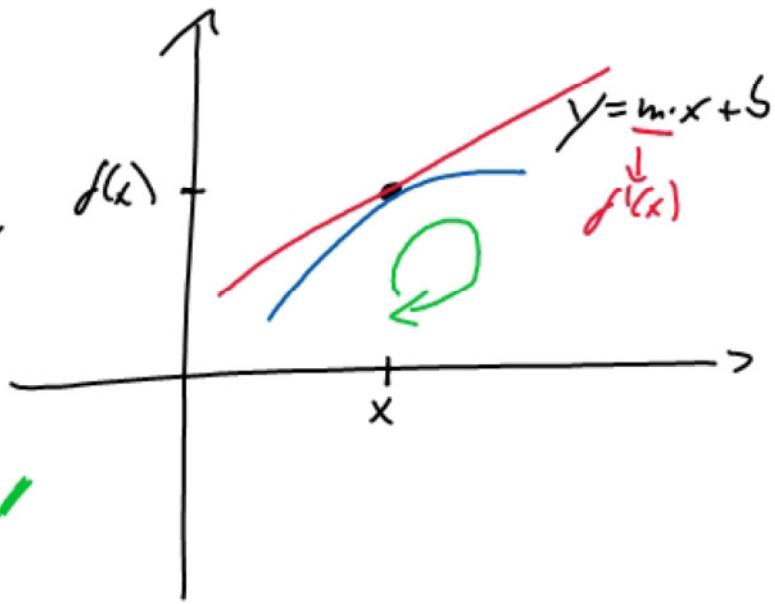
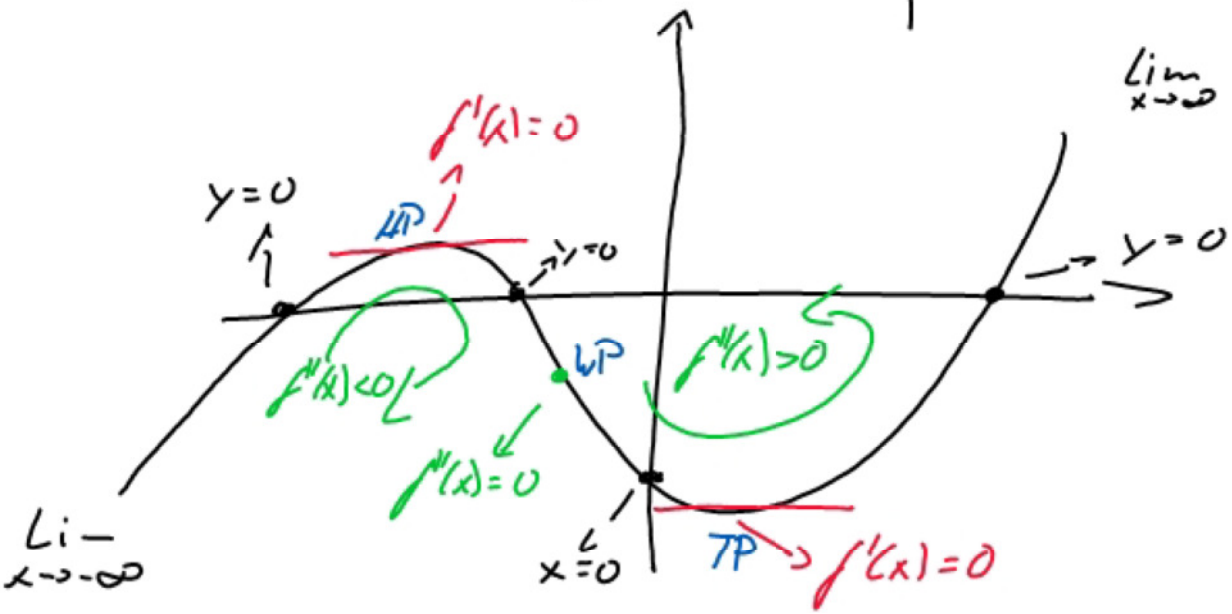
Funktionen

$f(x) \rightarrow$ y - Koordinate

$f'(x) \rightarrow$ Steigung

$f''(x) \rightarrow$ Krümmung





Ableitungen

$$f(x) = a \cdot x^n \quad \Rightarrow \quad f'(x) = a \cdot n \cdot x^{n-1}$$

$$f(x) = \tfrac{1}{2} \cdot x^4 \quad \Rightarrow \quad f'(x) = \tfrac{1}{2} \cdot 4 \cdot x^{4-1} = 2 \cdot x^3$$

Kettenregel $\quad f[g(x)] = f'[g(x)] \cdot g'(x)$

EXP: $\quad f(x) = e^{\heartsuit} \Rightarrow f'(x) = e^{\heartsuit} \cdot \heartsuit'$

$$f(x) = e^{3x} \rightarrow f'(x) = e^{3x} \cdot (3x)' = 3 \cdot e^{3x}$$

$$f(x) = e^{x} \rightarrow f'(x) = e^{x} \cdot (x)' = e^{x} \cdot 1 = e^{x}$$

LN :

$$f(x) = \ln \heartsuit \quad \Rightarrow \quad f'(x) = \frac{1}{\heartsuit} \cdot \heartsuit'$$

$$f(x) = \ln 3x^2 \qquad f'(x) = \frac{1}{3x^2} \cdot (3x^2)'$$

$$= \frac{1}{3x^2} \cdot 6x = \frac{2}{x}$$

Trigon :

$$f(x) = \sin \heartsuit \quad \Rightarrow \quad f'(x) = \cos \heartsuit \cdot \heartsuit'$$

$$f(x) = \cos \heartsuit \quad \Rightarrow \quad f'(x) = -\sin \heartsuit \cdot \heartsuit'$$

$$f(x) = \sin(7x) \quad \Rightarrow \quad f'(x) = \cos(7x) \cdot (7x)'$$

$$\cos(7x) \cdot 7$$

Potenz :

$$f(x) = \heartsuit^n \quad \Rightarrow \quad f'(x) = n \cdot \heartsuit^{n-1} \cdot \heartsuit'$$

$$f(x) = (2-3x^2)^4 \quad \Rightarrow \quad f'(x) = 4 \cdot (2-3x^2)^{4-1} \cdot (2-3x^2)'$$

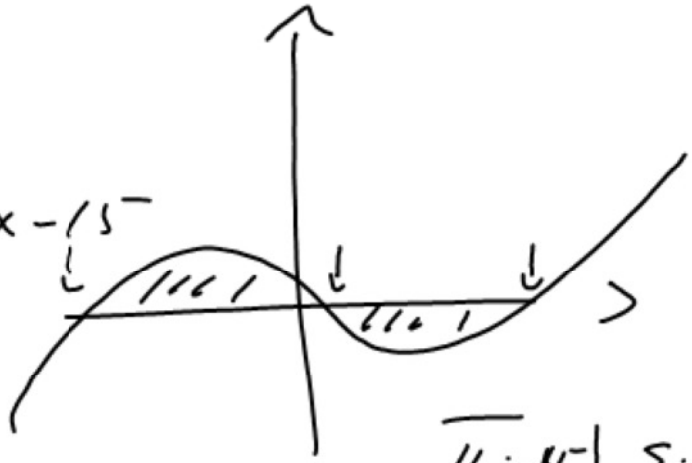
$$4 \cdot (2-3x^2)^3 \cdot (-6x)$$

$$f(x) = x^6 \quad \Rightarrow \quad f'(x) = 6 \cdot x^5 \, \underbrace{(x)'}_{1}$$

1) $x^3 - 4x^2 - 11x + 30 = 0$

$(x^3 - 4x^2 - 11x + 30)(x - 2) = x^2 - 2x - 15$

$$
\begin{array}{l}
-(x^3 - 2x^2) \\
\hline
\quad -2x^2 - 11x + 30 \\
\quad -(-2x^2 + 4x) \\
\hline
\qquad -15x + 30 \\
\qquad -(-15x + 30) \\
\hline
\qquad\quad - \quad -
\end{array}
$$



$x^2 - 2x - 15 = 0$

$\Pi: -15$	$\Sigma: -2$
$1 \quad -15$	✗
$-1 \quad 15$	✗
$3 \quad -5$	✓
$-3 \quad 5$	✗

$(x+3)(x-5)$

$\mathbb{L} = \{-3; 2; 5\}$

$\mathbb{L} = \{\pm 3; 7; -1\}$

2)

$$\frac{\frac{2}{9}+\frac{4}{15}}{\frac{4}{3}-\frac{7}{10}} = \frac{\frac{10+12}{45}}{\frac{40-21}{30}} = \frac{22}{\cancel{45}_{3}} \cdot \frac{\cancel{30}^{2}}{19} = \frac{44}{57}$$

$$\frac{\frac{3x}{4y}-\frac{5}{3z}}{\frac{5x}{6yz}+\frac{3z}{2x}} = \frac{\frac{9xz-20y}{12yz}}{\frac{5x^2+9yz^2}{6xyz}} = \frac{9xz-20y}{12\underset{2}{\cancel{yz}}} \cdot \frac{6x\cancel{yz}}{5x^2+9yz^2}$$

$$= \frac{9x^2z-20xy}{10x^2+18yz^2}$$

3) $\frac{2}{5x} - \frac{3}{4} + \frac{5}{12} - \frac{7}{6} = \frac{4}{15x} - \frac{9}{10} \quad | \cdot 60x$

$24 - 45x + 25x - 70x = 16 - 54x$

$24 - 90x = 16 - 54x \quad | + 90x - 16$

$8 = 36x \quad | : 36$

$x = \frac{8}{36} = \frac{2}{9}$

$$\sqrt[3]{x^5} = (x^5)^{1/3} = x^{5 \cdot 1/3} = x^{5/3}$$

$$x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

$$0{,}25^{-2} = \left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^{2} = 16$$

$$f(x) = \sqrt[3]{2x}$$

$$(2x)^{1/3}$$

$$f'(x) = \frac{1}{3} \cdot (2x)^{-2/3} \cdot 2$$

$$= \frac{2}{3 \cdot \sqrt[3]{(2x)^2}}$$

$$\frac{\sqrt[5]{x^{-3}}}{\left(\sqrt[3]{x}\right)^{-4}} = \frac{x^{-3/5}}{x^{-4/3}} = \frac{x^{4/3}}{x^{3/5}} = x^{4/3 - 3/5}$$

1) $x^2 \cdot \sqrt[3]{x \sqrt[4]{x^3 \cdot \sqrt{x}}}$

$\rightarrow x^2 \cdot \left(x \left(x^3 (x)^{1/2} \right)^{1/4} \right)^{1/3}$

$\rightarrow x^2 \; x^{1/3} \left(\left(x^3 \right)^{1/2} \right)^{1/3} \left(\left((x)^{1/2} \right)^{1/4} \right)^{1/3}$

$x^2 \cdot x^{1/3} \cdot x^{1/4} \cdot x^{1/24}$

$x^{\frac{48+8+6+1}{24}}$

$x^{\frac{63}{24}} = x^{21/8}$

2) $f(x) = \frac{\sqrt{x^3}}{x^2} \rightarrow f'(x)$

$\frac{\sqrt{x^3}}{x^2} = \frac{x^{3/2}}{x^2} = x^{3/2 - 2}$

$f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2} \cdot x^{-3/2}$

$= -\frac{1}{2 \cdot \sqrt{x^3}}$

3) $\left(\frac{1}{0{,}1^{-3}} \right)^2$

$\left[\frac{1}{\left(\frac{1}{10}\right)^{-3}} \right]^2 = \frac{1}{1\,\text{Mio}} = 10^{-6}$

1) $$\sqrt{x^3\sqrt[4]{x^6\sqrt[3]{x^2}}} = (x^3)^{1/2}\left(\left(x^6\right)^{1/4}\right)^{1/2}\left(\left(\left(x^2\right)^{1/3}\right)^{1/4}\right)^{1/2}$$

$$a^{\left(\frac{1}{k}\right)^{-2}} \qquad a^{k^2}$$

$$= x^{3/2}\cdot x^{3/4}\cdot x^{1/12}$$

$$= x^{\frac{18+9+1}{12}} = x^{28/12} = x^{7/3}$$

3) $$\frac{\sqrt[k]{a^{2-k}}}{\left(\sqrt[k]{a}\right)^{3k+4}}\left(\frac{\sqrt[k]{a}}{\left(\sqrt[k]{a^2}\right)^{k+3}}\right)^{-2} = \frac{a^{\frac{2-k}{k}}}{a^{\frac{3k+4}{k}}}\cdot\frac{a^{\frac{-2}{k}}}{a^{\frac{-4k-12}{k}}}$$

$$a^{\frac{2-k-(3k+4)+(-2)-(-4k-12)}{k}} = a^{8/k} = \sqrt[k]{a^8}$$