1)
$$\frac{-\frac{0{,}5}{5}-\frac{1}{2x_1}}{\frac{x_1}{5}+\frac{2}{1}+\frac{5}{x_1}} = \frac{\frac{-x_1-5}{10\,x_1}}{\frac{(x_1)^2+10x_1+25}{5x_1}}$$

$$\frac{-x_1-5}{\cancel{10}\,x_1} \cdot \frac{5x_1}{(x_1)^2+10x_1+25} = \frac{-(x_1+5)}{2\,(x_1+5)^2} = \frac{-1}{2\cdot(x_1+5)}$$

$$\frac{5}{2} \qquad (a^2+2ab+b^2)$$

5)
$$\frac{x^{1/2}\cdot\left(x^2\cdot\left(x^{1/2}\cdot x^5\right)^{1/3}\right)^{1/3}\cdot x^{1/9}}{x^2\cdot\left(x^5\cdot x^{2/3}\right)^{1/6}}$$

$$x^{1/2}\,x^{2/3}\,x^{1/12}\,x^{5/6}\,x^{1/9}\,x^{-2}\,x^{-5/6}\,x^{-1/9}$$

$$x^{\frac{6+8+1-24}{12}} \qquad x^{-9/12} = x^{-3/4} = \frac{1}{\sqrt[4]{x^3}}$$

c) $$\sqrt[3k]{\left(z^{k-3}\right)^{6}} \cdot \frac{\left(\sqrt[3]{\sqrt[k]{z^{3}}}\right)^{3-2k}}{\sqrt[4k]{z^{4k-12}}}$$

$$z^{\frac{(k-3)\cdot 6}{3k}} \cdot \frac{z^{\frac{3\cdot(3-2k)}{3k}}}{z^{\frac{4k-12}{4k}}} = \frac{4\cdot(k-3)}{4k}$$

$$z^{\frac{2\cdot(k-3)+(3-2k)-(k-3)}{k}}$$

$$z^{\frac{-k}{k}} = z^{-1} = 1/z$$

d) $$\frac{3^2 \cdot (2^{-1} \cdot x^2 \cdot y^{-2} z)^4 \cdot 2^4 (3 \cdot x^4 y^3 z^{-4})^3}{3^3 \cdot 2 \cdot (2^2 \cdot x^{-2} y^3 z^{-2})^{-3} \cdot 3^2 \cdot 2^2 \cdot (2 \cdot x^2 y^5 z^{-4})^2}$$

$$\frac{3^2\, 2^{-4}\, x^8\, y^{-8}\, z^4\, 2^4\, 3^3\, x^{12}\, y^9\, z^{-12}}{3^3\, 2\, 2^{-6}\, x^6\, y^{-9}\, z^6\, 3^2\, 2^2\, 2^2\, x^4\, y^{10}\, z^{-8}}$$

$$\frac{3^2\, 2^4\, 3^3\, 2^6}{2^4\, 2\, 3^2\, 2^2\, 2^2\, 3^3} \qquad \frac{y^8\, z^4\, x^{12}\, y^9\, y^9\, z^8}{y^8\, z^{12}\, x^6\, z^6\, x^4\, y^{10}}$$

$$2 \cdot \frac{x^{10}}{z^6} = 2\, x^{10} z^{-6}$$

$$f(x) = \sqrt[3]{\frac{3}{x-2}}$$

→ $\mathbb{D}$ und $\mathbb{W}$

→ Achsenschnittpunkte

→ Skizze

→ Warum existieren keine Extrema?

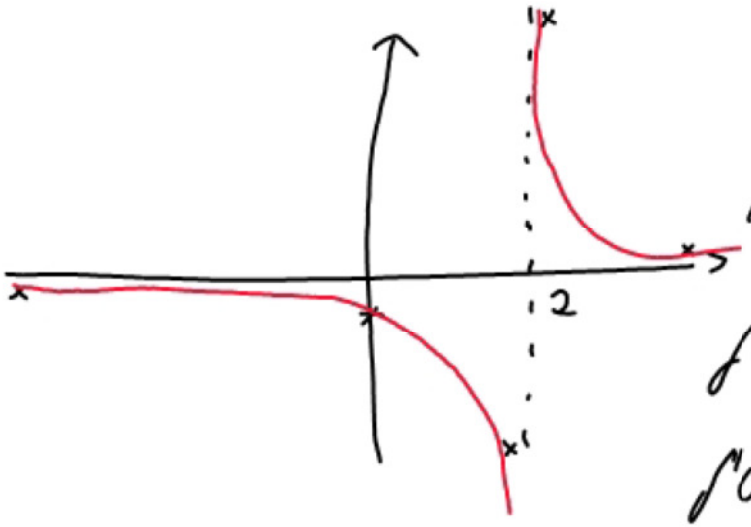
$$\mathbb{D} = x \in \mathbb{R} \setminus \{2\}$$

$$f(0) = \sqrt[3]{3/-2} \approx -1$$

$$f(x) = 0 = \sqrt[3]{\frac{3}{x-2}} \quad | \uparrow^3$$

$$0 = 3/x-2 \quad | \cdot x-2$$

$$0 = 3 \quad ↯$$



$$\mathbb{W} = y \in \mathbb{R} \setminus \{0\}$$

$$f(x) = \sqrt[3]{3} \cdot (x-2)^{-1/3}$$

$$f'(x) = \sqrt[3]{3} \cdot (-1/3) \cdot (x-2)^{-4/3}$$

$$= -\frac{\sqrt[3]{3}}{3} \cdot \frac{1}{\sqrt[3]{(x-2)^4}}$$

Logarithmus

$2^x = 32 = 2^5$ Basisvergleich: $x = 5$

$2^x = 25$ $x \in \,]4;5[$

$a^x = b \iff x = \log_a b = \frac{\log b}{\log a}$

$\rightarrow x = \log_2 25 = \mathrm{ld}\, 25$

$\log_5(<0) = x \iff 5^x < 0$

$\log_5(0) = x \iff 5^x = 0$

$\mathbb{D}_{\log} = \mathbb{R}^{>0}$

1) $3 \cdot \log(x-y) + \log(x+y) - \frac{1}{2} \cdot \log(x-y)^4$

$\log(x-y)^3 + \log(x+y) - \log\left[(x-y)^4\right]^{1/2}$

$\log a + \log b - \log c$

$\log \frac{(x-y)^3 \cdot (x+y)}{(x-y)^2} = \log(x-y)(x+y) = \log(x^2-y^2)$

2) $2\ln 2x - 3\ln 2 + 4\ln\sqrt{x} + 2\ln \frac{4}{x^2}$

$\ln(4x^2) - \ln 8 + \ln x^2 + \ln \frac{16}{x^4}$

$\ln \frac{4x^2 \; x^2 \; \frac{16}{x^4}}{8} = \ln 8$

3) $\log \sqrt[5]{\dfrac{x^3 y^2}{3\cdot(x+y^2)}} = \frac{1}{5}\cdot \log \dfrac{x^3 y^2}{3\cdot(x+y^2)}$

$\left[\quad + \quad \right]$

$\frac{1}{5}\cdot\left[\log x^3 + \log y^2 - \log 3 - \log(x+y^2)\right]$

$\frac{3}{5}\log x + \frac{2}{5}\log y - \frac{1}{5}\log 3 - \frac{1}{5}\log(x+y^2)$

4) $\ln\left(\dfrac{2\sqrt{a-2b}}{c^2\sqrt[4]{d}}\right)^3 = 3\cdot \ln\left(\dfrac{2\sqrt{a-2b}}{c^2\sqrt[4]{d}}\right)$

$3\left[\ln 2 + \ln(a-2b)^{1/2} - \ln c^2 - \ln d^{1/4}\right]$

$3\ln 2 + \frac{3}{2}\cdot\ln(a-2b) - 6\ln c - \frac{3}{4}\ln d$