1) $R = \{ (a,b) \in M \times M \mid r = \frac{a+1}{b+1} \;;\; r \in \mathbb{N} \}$

<u>reflexiv</u> : $(a,a) \in R \;;\; a \in M$

$$r = \frac{a+1}{a+1} = 1 \in \mathbb{N} \quad \checkmark$$

<u>transitiv</u> $(a,b) \in R \land (b,c) \in R \Rightarrow (a,c) \in R$

$$r_1 = \frac{a+1}{b+1} \land r_2 = \frac{b+1}{c+1} \Rightarrow \frac{a+1}{c+1} = r_3$$

$$b+1 = r_2 \cdot (c+1)$$

$$r_1 = \frac{a+1}{(c+1) \cdot r_2} \quad | \cdot r_2$$

$$r_1 \cdot r_2 = \frac{a+1}{c+1}$$

$\mathbb{N} \cdot \mathbb{N}$ $\quad \mathbb{N}$

Symmetrie $(999, 99) \to \frac{999+1}{99+1} = 10 \in \mathbb{N}$

$\frac{100}{1000} \notin \mathbb{N}$

antisymmetrie: $(a,b) \in R \wedge (b,a) \in R$;
zeige $a = b$

$\frac{a+1}{b+1} = r_1 \wedge \frac{b+1}{a+1} = r_2 \quad | \cdot (a+1)$

$b+1 = r_2 \cdot (a+1)$

$\frac{a+1}{r_2 \cdot (a+1)} = r_1$

$\frac{1}{r_2} = r_1 \Rightarrow r_1 = r_2 = 1 \to (a,a) \in R$

$\Rightarrow$ Ordnungsrelation

3) a)

$$\lim_{x \to (-3)} \frac{\overbrace{5x + 15}^{\to\ 5\cdot(x+3)}}{\underbrace{\sqrt{7-3x} - (1-x)}} \cdot \frac{\sqrt{7-3x} + (1-x)}{\sqrt{7-3x} + (1-x)}$$

$$(7-3x) - (1-x)^2 = 7-3x-x^2+2x-1$$

$$-x^2 - x + 6$$

$$-(x^2+x-6) = -(x+3)(x-2)$$

$$\lim_{x \to (-3)} \frac{5\cdot(\sqrt{7-3x} + (1-x))}{-(x-2)} = \frac{5\cdot(4+4)}{5} = 8$$

b)

$$\lim_{x \to (-3)} \frac{5}{\frac{1}{2\cdot\sqrt{7-3x}}\cdot(-3) + 1} = \frac{5}{-\frac{3}{8}+1} = 8$$

4) $n^2 - 2n - 1 > 0 \quad ; n \geq 3$

$n = 3 \quad 9 - 6 - 1 = 2 > 0$ ✓

Prämisse $n^2 - 2n - 1 > 0$ für alle $n \geq 3$

$n+1 \quad (n+1)^2 - 2\cdot(n+1) - 1 > 0$

$n^2 + 2n + 1 - 2n - 2 - 1 > 0$

$$\underbrace{n^2 - 2n - 1}_{>0} + \underbrace{(2n - 1)}_{>0}$$

$2n - 1 > 0 \quad ; n \geq 3$

⋮

5) $f(x) = \begin{cases} \sqrt{ax} & ; x < a \\ a \cdot x^2 + s & ; x \geq a \end{cases}$

$f'(x) = \begin{cases} \frac{1}{2 \cdot \sqrt{ax}} \cdot a & ; x < a \\ 2ax & ; x \geq a \end{cases}$

$s = 3/8$

$1/2 = 1/8 + s$

$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

$a^3 + s = \sqrt{a^2} = a^3 + s \Rightarrow a = a^3 + s$

$a = 1/2$

$\lim_{x \to a^+} f'(x) = \lim_{x \to a^-} f'(x) = f'(a)$

$2a^2 = \frac{a}{2\sqrt{a^2}} = 2a^2 \Rightarrow \frac{1}{2} = 2 \cdot a^2 \; ; \; a^2 = 1/4$

$a = \pm 1/2$

6)

$$\int_0^t \underbrace{\left(x^2 - \tfrac{2}{3}t \cdot x + 2\right)}_{} dx = F(t) - F(0) = 84$$

$$F(x) = \tfrac{1}{3}x^3 - \tfrac{2}{3}t \cdot \tfrac{1}{2}x^2 + 2x = \tfrac{1}{3}x^3 - \tfrac{1}{3}tx^2 + 2x$$

$$F(t) = \underbrace{\tfrac{1}{3}t^3 - \tfrac{1}{3}t \cdot t^2}_{0} + 2t = 2t$$

$$F(0) = 0$$

$$2t = 84 \qquad t = 42$$

7) $a_{n+1} = \frac{2}{3} a_n + \frac{10}{3}$ $\quad a_1 = 20 \; ; \; a_2 = 16\frac{2}{3}$

Behauptung $\quad a_{n+1} < a_n \quad$ (fallend

$n=1 \quad a_2 < a_1 \quad 16\frac{2}{3} < 20$ ✓

Prämisse: Die Folge fällt und es gilt $a_{n+1} < a_n$

$n+1 \quad a_{n+2} < a_{n+1}$

$$\frac{2}{3} a_{n+1} + \frac{10}{3} < \frac{2}{3} a_n + \frac{10}{3} \quad \Big| -\frac{10}{3}$$

$$\frac{2}{3} a_{n+1} < \frac{2}{3} a_n \quad \Big| \cdot \frac{3}{2}$$

$$a_{n+1} < a_n$$ ✓

<u>Schranken</u>: Da a_n streng monoton fallend ist, muss $a_1 = 20$ obere Schranke sein.

Behauptung $\quad a_n > 10$

$n=1 \qquad a_1 = 20 > 10 \quad \checkmark$

$n+1 \qquad a_n > 10 \qquad | \cdot \frac{2}{3}$

$$\frac{2}{3} a_n > \frac{20}{3} \qquad | + \frac{10}{3}$$

$$\underbrace{\frac{2}{3} a_n + \frac{10}{3}}_{} > \frac{20}{3} + \frac{10}{3} = 10$$

$$a_{n+1} > 10 \quad \checkmark$$

<u>Grenzwert</u>:

$$\lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} a_n$$

$$\lim_{n \to \infty} a_n = r \qquad a_{n+1} = \frac{2}{3} a_n + \frac{10}{3}$$

$$r = \frac{2}{3} \cdot r + \frac{10}{3} \qquad \Big| - \frac{2}{3} r$$

$$\frac{1}{3} r = \frac{10}{3} \qquad \Big| \cdot 3$$

$$\underline{\underline{r = 10}}$$

g) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

NS : $f(1) = 0$ $\quad a + b + c + d = 0$

WP $\quad f(2) = 14$ $\quad 8a + 4b + 2c + d = 14$

$f''(2) = 0$ $\quad 12a + 2b = 0$

$t(x)$ $\quad f'(2) = 15$ $\quad 12a + 4b + c = 15$

$$
\left|\begin{array}{l}
a + b + c + d = 0 \\
8a + 4b + 2c + d = 14 \\
12a + 4b + c = 15 \\
12a + 2b = 0
\end{array}\right| \ominus
$$

$$
\left|\begin{array}{l}
a + b + c + d = 0 \\
-7a - 3b - c = -14 \\
12a + 4b + c = 15 \\
12a + 2b = 0
\end{array}\right| +
\qquad
\cdot(-2) \left|\begin{array}{l}
a + b + c + d = 0 \\
-7a - 3b - c = -14 \\
5a + b = 1 \\
12a + 2b = 0
\end{array}\right|
$$

$$
\left|\begin{array}{l}
a + b + c + d = 0 \\
-7a - 3b - c = -14 \\
5a + b = 1 \\
2a = -2
\end{array}\right|
$$

$d = -8$

$c = 3$

$b = 6$

$a = -1$

$\rightarrow 7 - 18 - c = -14$

$$a + b + c + d = 0$$
$$8a + 4b + 2c + d = 14$$
$$12a + 4b + c = 15$$

$$b = -6a$$

$$-5a + c + d = 0$$
$$-16a + 2c + d = 14$$
$$-12a + c = 15 \quad \rightarrow \quad c = 15 + 12a$$

$$-5a + (15 + 12a) + d = 7a + 15 + d = 0$$
$$-16a + 2\cdot(15 + 12a) + d = -16a + 30 + 24a + d = 14$$
$$8a + d = -16$$

$$\left| \begin{array}{l} 7a + d = -15 \\ 8a + d = -16 \end{array} \right| \Big\} \ominus \qquad -a = 1$$

$$\rightarrow 7a + d = -15$$
$$d = -8$$

$$a = -1$$
$$b = 6$$
$$c = 3$$