Klausur 2016/17

4)

$$\underset{\substack{\uparrow\\ 1.}}{1} + \underset{\substack{\uparrow\\ 2.}}{4} + \underset{\substack{\uparrow\\ 3.}}{7} + \dots + (\underbrace{3n-2}_{n.}) = \frac{1}{2}n\cdot(3n-1)\,;\quad n\geq 1$$

$$\sum_{k=1}^{n}(\underbrace{3k-2}_{a_k}) = \underbrace{\frac{1}{2}n\cdot(3n-1)}_{S_n}$$

<u>$n=1$</u> $\quad a_1 = S_1 \qquad (3\cdot 1-2) = 1 = \frac{1}{2}\,1\cdot(3\cdot 1-1)$ ✓

<u>Prämisse</u>: Der Ausdruck $\sum_{k=1}^{n}(3k-2) = \frac{1}{2}n\cdot(3n-1)$ ist für alle $n \geq 1$ gültig.

$\underline{n+1}$

$$\underbrace{\sum_{k=1}^{n+1}(3k-2)}_{S_{n+1}} = \underbrace{\sum_{k=1}^{n}(3k-2)}_{S_n} + a_{n+1}$$

$$S_{n+1} = S_n + a_{n+1}$$

$$\tfrac{1}{2}(n+1)\cdot[3(n+1)-1] = \tfrac{1}{2}\cdot n\cdot(3n-1) + [3\cdot(n+1)-2] \quad |\cdot 2$$

$$(n+1)\cdot(3n+2) = n\cdot(3n-1) + 2\cdot(3n+1)$$

$$3n^2+\underbrace{3n+2n}_{5n}+2 = 3n^2\underbrace{-n+6n}_{5n}+2 \quad |-3n^2-5n-2$$

$$0=0 \quad \checkmark$$

8) a) $\frac{1}{2} \cdot \sum_{k=3}^{\infty} \frac{2^{k+2}}{k!} = \frac{1}{2} \cdot \sum_{k=3}^{\infty} \frac{2^{k} \cdot 2^{2}}{k!} = 2 \cdot \sum_{k=3}^{\infty} \frac{2^{k}}{k!}$

$\sum_{k=0}^{\infty} \frac{x^{k}}{k!} = e^{x}$

$2 \cdot \left[\sum_{k=0}^{\infty} \frac{2^{k}}{k!} - \left(\frac{2^{0}}{0!} + \frac{2^{1}}{1!} + \frac{2^{2}}{2!} \right) \right]$

$2 \cdot \left(e^{2} - (1 + 2 + 2) \right] = 2 e^{2} - 10$

b) $1024 \cdot \sum_{k=1}^{4} \left(\frac{1}{2}\right)^{2\cdot(2+k)} = 2^{10} \cdot \sum_{k=1}^{4} \left(\frac{1}{2}\right)^{4} \cdot \left(\frac{1}{2}\right)^{2k}$

$2^{6} \cdot \sum_{k=1}^{4} \left(\frac{1}{4}\right)^{k} = 2^{6} \cdot \sum_{k=0}^{4} \left(\frac{1}{4}\right)^{k+1} = 2^{4} \cdot \sum_{k=0}^{4} \left(\frac{1}{4}\right)^{k}$

$\sum_{k=0}^{n} q^{k} = \frac{1 - q^{n+1}}{1 - q}$

$\Rightarrow 2^{4} \cdot \frac{1 - \left(\frac{1}{4}\right)^{5}}{1 - \frac{1}{4}} = \underset{\downarrow \; 64/3}{\frac{2^{6}}{3}} \cdot \left(1 - \left(\frac{1}{4}\right)^{5}\right)$

$$a_{n+1} = \sqrt{6 + 5a_n} \; ; \; a_1 = 2 \quad , n \geq 1$$

$$a_2 = \sqrt{6 + 5\, a_1} = \sqrt{16} = 4$$

$$a_3 = \sqrt{6 + 5 \cdot a_2} = \sqrt{26} \approx 5$$

$a_{n+1} > a_n$

$\Downarrow$

Behauptung

Monotonie: $a_{n+1} > a_n$

$n = 1 \qquad a_2 > a_1 \qquad \Rightarrow \qquad 4 > 2 \qquad \checkmark$

Prämisse: Die Folge a_{n+1} ist streng monoton steigend und es gilt $a_{n+1} > a_n$

$n+1$: $\quad a_{n+2} > a_{n+1} \quad : \quad \sqrt{6 + 5 \cdot a_{n+1}} > \sqrt{6 + 5a_n} \quad |\uparrow^2$

$$6 + 5a_{n+1} > 6 + 5a_n \quad |-6 \mid \cdot \tfrac{1}{5}$$

$$a_{n+1} > a_n \quad \checkmark$$

Schranke: Da a_{n+1} streng monoton steigend ist,

muss $a_1 = 2$ untere Schranke sein

$a_n < 6$ $\quad n=1 \quad a_1 < 6 \Rightarrow 2 < 6$ ✓

Prämisse: 6 ist obere Schranke, so dass $a_n < 6$ gültig ist.

$a_n < 6 \quad | \cdot 5 \quad \Leftrightarrow \quad 5 \cdot a_n < 6 \cdot 5 = 30 \quad | +6$

$6 + 5a_n < 30 + 6 = 36 \quad | \sqrt{}$

$\sqrt{6 + 5 \cdot a_n} < \sqrt{36} \quad \Rightarrow \quad a_{n+1} < 6$

Grenzwert:

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n = r$$

$$\sqrt{6+5r} = r \quad |()^2$$

$$6+5r = r^2 \quad |-5r-6$$

$$r^2-5r-6 = (r-6)\cdot(r+1) = 0$$

$$r=-1 \quad \vee \quad r=6$$

$\mathbb{L}=\{6\}$, da $a_1=2$ untere Schranke gilt.