S 140 Nr. 4

$$4 \cdot \sum_{k=3}^{8} 4 \cdot \left(\tfrac{1}{2}\right)^{2k} = 16 \cdot \sum_{k=3}^{8} \underbrace{\left(\tfrac{1}{2}\right)^{2k}}_{q^k} = 16 \cdot \sum_{k=3}^{8} \left[\left(\tfrac{1}{2}\right)^{2}\right]^{k}$$

$$\Rightarrow 16 \cdot \sum_{k=3}^{8} \left(\tfrac{1}{4}\right)^{k}$$

$$\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$$

4^2

$$16 \cdot \sum_{k=0}^{8} \left(\tfrac{1}{4}\right)^{k+3} = 16 \cdot \sum_{k=0}^{8} \left(\tfrac{1}{4}\right)^{3} \cdot \left(\tfrac{1}{4}\right)^{k}$$

$$\tfrac{1}{4} \cdot \sum_{k=0}^{8} \left(\tfrac{1}{4}\right)^{k} = \frac{1}{4} \cdot \left[\frac{1-\left(\tfrac{1}{4}\right)^{9}}{\underbrace{1-\tfrac{1}{4}}_{3/4}}\right] = \tfrac{1}{3} \cdot \left(1-\left(\tfrac{1}{4}\right)^{9}\right)$$

$x^{3+5} = x^3 \cdot x^5$

5) $\sum_{k=4}^{\infty} \left(1/3\right)^{2k+1} = \sum_{k=4}^{\infty} \left[\left(1/3\right)^2\right]^k \cdot \left(1/3\right)^1$

$$1/3 \cdot \sum_{k=4}^{\infty} \left(1/9\right)^k = 1/3 \cdot \sum_{k=0}^{\infty} \left(1/9\right)^{k+4} = \frac{1}{3} \cdot \left(\frac{1}{9}\right)^4 \cdot \sum_{k=0}^{\infty} \left(1/9\right)^k$$

$$\left(1/3\right)^{2k} \cdot 1/3^{\,1}$$

$$\frac{1}{3} \cdot \frac{1}{9^4} \cdot \frac{1}{1 - 1/9} = \frac{1}{3} \cdot \frac{1}{9^4} \cdot \frac{9}{8} = \frac{1}{8 \cdot 3^7}$$

$$6)\quad \sum_{k=2}^{\infty} (-1)^k \cdot \frac{3 \cdot \pi^{2k}}{(2k)!} \qquad ; \qquad x = \pi$$

$$3 \cdot \sum_{k=2}^{\infty} (-1)^k \cdot \frac{\pi^{2k}}{(2k)!}$$

$$3 \cdot \left[\sum_{k=0}^{\infty} (-1)^k \cdot \frac{\pi^{2k}}{(2k)!} - \left((-1)^0 \cdot \frac{\pi^{2\cdot 0}}{(2\cdot 0)!} + (-1)^1 \cdot \frac{\pi^{2\cdot 1}}{(2\cdot 1)!} \right) \right]$$

$$3 \cdot \left[\cos \pi - \left(1 - \tfrac{1}{2} \pi^2 \right) \right]$$

$$-6 + \tfrac{3}{2} \pi^2$$

S 198 Zusatz: für welches x konvergiert:

$$\sum_{k=1}^{\infty} \frac{(-1)^{3k+1} \cdot (5x)^{2k}}{2^{4k} \cdot k^2} = \underbrace{\sum_{k=1}^{\infty} (-1)^{3k+1}}_{\text{Alternierungswert}} \cdot \frac{(5x)^{2k}}{2^{4k} \cdot k^2}$$

Leibniz: $\lim\limits_{k \to \infty} \frac{(5x)^{2k}}{2^{4k} \cdot k^2} = 0$

$$\lim_{k \to \infty} \underbrace{\left[\frac{5x}{4}\right]^{2k}}_{\text{Nullfolge}} \cdot \underbrace{\frac{1}{k^2}}_{\text{Nullfolge}} \qquad x \in \left[-\tfrac{4}{5} ; \tfrac{4}{5}\right]$$

$$\frac{(5x)^{2k}}{2^{2 \cdot 2k}} = \frac{(5x)^{2k}}{(2^2)^{2k}} =$$